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THE LAWS OF GROWTH OF A POPULATION

By Sir George H. Knibbs, C. M. G.

PART I

Synopsis: 1. Introduction. 2. Normal law of population growth. 3. Factors determining rate of increase. 4. Nature of varying increase. 5. The theory of the "logistic" curve. 6. Skew and other forms of the logistic curve. 7. Assumptions under which populations can conform to the logistic curve. 8. Tests as to the legitimacy of fitting a logistic curve to population results. 9. Population curve of the United States. 10. What governed the rates of increase in the United States. 11. Is the population of France consistent with the logistic theory? 12. Population increase with a fixed area. 13. Supposed conditions of the geometric growth of a population. 14. The secular growth of populations. 15. Fitting power of logistic curve not proof of the theory. 16. Causes of automatic variations of rate of increase. 17. Effect of the productivity of a territory. 18. Correlation of productivity, marriage and birth-rates and rate of increase. 19. Population density and its effect on increase. 20. Effect of war on birth-rate. 21. Effect of economic disaster on rate of increase. 22. Conclusions.

1. Introduction. The laws of growth of human populations have recently occupied special attention. In Professor Raymond Pearl's book [1] the whole question has been reviewed very comprehensively and ably from the standpoint of exhaustion of the population-carrying power of any territory through the growth of population itself. Empirically it had been shown by A. S. Pritchett in 1891 that an equation of the form

$$(1) \quad P = a + bt + ct^2 + dt^3$$

would fit the actual curve fairly well[2]; and by Pearl himself,[3] that a formula of the type

$$(2) \quad P = a + bt + ct^2 + d \log t$$

[1] *Studies in Human Biology*, Ch. XXIV, "The Curve of Population Growth," 1924.
[2] This Journal, Vol. 2, 1891, pp. 278-286.
[3] Carnegie Institute Washington Publication, Vol. 58, 1907.

had great fitting power. These expressions are, however, really only empirical. More recently G. Udny Yule has reopened the question,[1] but it has been pointed out by Professor A. L. Bowley that "there is . . . no reason *a priori* to suppose that the damping down is of so regular or uniform a nature, that a mathematical function of the same form represents it at all times and in all places." [2]

It is proposed in this article to ask what are the laws actually operating, and to show that what Verhulst [3] has called the logistic curve does *not* very accurately represent rationally the facts for human populations, although the curve may be empirically fitted to the actual facts, approximately. It is believed that the deductions as to the actual limiting values of population are very precarious, and that an exact study of the facts will show this.

2. Normal law of population growth. When a *considerable* population, viz., one sufficiently large to be relatively unaffected by changes at any age of any small number of units, continuously occupies a territory, then so long as its constitution as to sex and age, and the elements of its physical environment, and of its internal impulses and character, remain sensibly constant, the ratio of its decreases through deaths and of its increases through births, will also remain sensibly constant. If, through this constancy, it has reached a stable state as regards external environment and internal characters [4], it will then increase, sensibly always in the same definite ratio to its total number. Thus if P denote this number, and r be the *rate* of increase at every instant of its growth, per unit of time (say per annum), then after the time t it will have become Pe^{rt}, that is it will increase exactly as a sum increases at compound interest.

The relative measure of the incidence of death and the relative measure of actual reproduction, each sensibly uniform for any particular age, are assumed to be such that the circumstances of birth and death do not alter the constitution according to sex and age; that is to say, the "make-up" of the population in this respect is assumed not to alter. The tendency to increase according to the compound-interest law may

[1] See *Journal of the Royal Statistical Society*, January, 1925, pp. 1–62.

[2] *Ibid.*, p. 76.

[3] Notice sur la loi, que la population suit dans son accroissement Correspondence mathématique et physique publiée par A. Quetelet, Tome X, 1838, pp. 113–121. Also Recherches mathématiques sur la loi d'accroissement de la population. Nouveaux Mémoires de l'Académie Royal de Science et Belles-Lettres de Bruxelles, Tome XVIII, 1845, pp. 1–38. Also Deuxième Mémoires sur la loi etc., Tome XX, 1847, pp. 1–32.

[4] In such a case the groups being added through the effluxion of time are the remanets at each age from births from a previous date: thus the group appearing in age 30 to age 31 is that proportion only of the total births 30 to 31 years earlier that has remained alive for the period in question. This may modify the constitution of the population according to age, but the consequence of this will be considered later.

be regarded as *approximately* the normal law of the increase of all populations at any given moment. But it can be regarded only as the instantaneous law: it cannot properly be assumed that the rate of increase will remain constant, for reasons to which we shall now refer.

3. Factors determining rate of increase. The main elements causing an increase of the rate of growth are the following:

(a) The utilizable physical resources of a population including the territory occupied by it;
(b) The skill with which these are exploited;
(c) The measure of the population's response to the reproductive impulse;
(d) The reduction of its death-rates; and
(e) The reduction of its rate of emigration.

Similarly the main elements causing a decrease of the rate of growth are the following:

(f) The standard adopted in the satisfaction of their needs;
(g) The diminution of utilizable physical resources;
(h) Ignorance as to making use of these;
(i) Impairment of the reproductive impulse;
(j) Increase of death-rates; and
(k) Increase of the rate of emigration.

The above (a) to (k) are of course only the principal factors, the resultant of which determines the rate at which a population grows. Since all of these have an annual periodicity, should units of time smaller than one year be taken into account, then the growth of population is subject to many minor fluctuations. For the main features of the general theory, however, these minor fluctuations may be disregarded, and only the general trend for each year need be considered.

4. Nature of varying increase. Whatever be the country under review the population is never found to increase at a uniform rate; whether the values of this rate be known annually as in a few countries, or quinquennially or decennially only, as in most. For convenience it may be assumed that the rate does not change suddenly, but changes gradually. Though this is only a minor point it is worthy of attention. Thus if for one quinquennium a population increased at the rate of say 2.6 per cent (i.e., if r_1 was say 0.026 per 5 years) and if for the next 5 the rate of increase was say 3.0 per cent (i.e., if r_2 was 0.030 per 5 years), we might very properly adopt *any plausible assumption* as to the mode of increase. Thus in the absence of any other evidence, for example, we might assume that initially the rate probably was less than 0.026, that at the end of the first period it was between 0.026 and 0.030, and finally that probably it was more than 0.030.

If the initial population was say 1,000,000; at the end of the first quinquennium it would be 1,026,000 and at the end of the second this number multiplied by 1.030, viz., 1,056,780, according to the rates mentioned.

For population to change in the above uniform and simple way so as to give these figures, we could quite appropriately adopt the successive ratios: R, Rm, Rm^2, Rm^3, etc., and readily find $R=1.004834$ and $m=1.0001556$. In this way we would get the results in column (I) hereunder. Or we could as plausibly assume that the populations for the successive years were on a parabolic curve given by $P=1{,}000{,}000+4722t+95.6t^2$, and thus obtain the results in column (II).

Years elapsed	(I)	(II)	Years elapsed	(I)	(II)
1	1,004,834	1,004,818	6	1,031,762	1,031,774
2	1,009,848	1,009,826	7	1,037,718	1,037,739
3	1,015,046	1,015,026	8	1,043,871	1,043,895
4	1,020,429	1,020,417	9	1,050,224	1,050,242
5	1,026,000	1,026,000	10	1,056,780	1,056,780

The greatest difference between these is 24; of course neither interpolation is anything more than a probable value; the results in column (I) simply assume that in each successive year the rate of growth increases by a common factor 1.0001556; those in column (II) assume, merely, that the second difference in the population numbers is constant, viz., a little over 191 (about 191.2), that is to say that the numbers for the whole series lie on the parabolic curve.

As a matter of fact populations do not grow in these regular ways. When conditions generally are favorable to increases of population, the rate of increase becomes greater, and as soon as the circumstances are adverse it decreases. So that for intermediate years the populations are supposititious only, however calculated; it is merely for reasons of convenience that they are interpolated in a simple way, not according to a supposed real probability of the case, which as a rule is not known. The *relative amount of the error* is, however, probably not great, and for practical purposes may be negligible.

We shall now consider the nature of the increase when the impulse to increase may be assumed to have remained constant, but for the fact that ultimately it must be limited by the reduction of the extent of the environment per individual.

5. The theory of the logistic curve. If the *relative change* in any population, that is the ratio of $\Delta y/\Delta t$ to the population y, be supposed to be always proportional to the *relative remaining possibility of population*, that is to the ratio of $(L-y)$ to the ultimate limit of population,

L say, it will become zero when the population reaches the assumed limit L. This assumption is expressed by the differential equation

$$(3) \quad \frac{dy}{dt} \Big/ y = r(1-y/L)$$

in which r represents the rate of increase, supposed to be constant throughout, but effective only in the measure of the factor $(L-y)/L$. The solution of this differential equation may be written

$$(4) \quad y = L/[1+e^{r(b-t)}]$$

b being a constant of integration. This curve has been called the logistic curve.

If t be infinite, $y=L$; if $t=b$, then $y=\frac{1}{2}L$. Taking the second differential, we have

$$(5) \quad d^2y/dt^2 = r(1-2y/L)$$

which is zero if $y=\frac{1}{2}L$. Thus, when $t=b$, we have the point of inflexion. For equal values of t on either side of this point, the values of y differ from $\frac{1}{2}L$ by equal amounts, that is to say the curve is symmetrical about the point of inflexion. Moreover, when y/L is negligible as compared with unity, we have $dy/y=r$, hence $\log_e y = rt+C$; thus $y=Ke^{rt+C}$, that is to say the curve is a geometric progression: its increase is as in compound interest.

This quantity r, viz., the increase per unit of population per unit of time, may be regarded as *the normal impulse to increase* when such impulse can have its full effect, when it is not hindered by limitations arising from a sensible reduction of the remaining relative possibility of population, $(L-y)/L$ in such a case being very nearly unity. Or, to express it otherwise, the inherent impulse to increase may be regarded as constant, excepting in so far as it is made inoperative through increase in the density of population.

A very little consideration will show that, within restricted regions, the application of equation (4) to the development of bacteria and in the entomological field, are presumably more valid than in the field of human populations, because in the latter case the reproductive and other impulses to increase are subject to modification not only from economic and regional restrictions, but also those arising from changes in the point of view, and in the whole external environment.

6. Skew and other forms of the logistic curve. The curve is not necessarily symmetrical. Professor Pearl has considered the question of possible variations in the form of the curve; reference may be made to his work for their development.[1] It will suffice here to remark that they can be subsumed under the form of

[1] Pearl, *op. cit.* pp. 569–576. See Figs. 99 to 103 and formula (XIX) p. 575.

$$(6) \quad y=A+K/[1+e^{F(x)}]$$

A curve may be built up by a series of additions. Thus it is not essential that there should be symmetry on either side of the point or points of inflexion.

7. Assumptions under which populations can conform to the logistic curve. The fundamental assumptions underlying the conception that a population will develop conformably to the logistic curve, are as follows: (a) It is associated with a restricted area; (b) the possibility of increase is diminished in the ratio of the remaining possibility-of-population to the total possible-population (thus that the inherent impulse to increase has its maximum value initially); (c) the increment of population is always proportional to the total population at the instant considered but the proportion changes; (d) the inherent impulse to increase is modified solely by the fact that the relative possible increase has become smaller, and not by other additive restrictions.

In regard to these it ought primarily to be observed that *a small population in a very large area,* say one of four millions in an area capable of carrying 200 millions can, as a matter of fact, probably expand for a considerable time in response solely to its inherent impulse to increase, since all limitations of its physical environment are really unchanged by its growth. In such a case it can initially be regarded as occupying an indefinitely large, or unrestricted, area. If one finds therefore that, as the ratio of the total population to the total possible-population passes through a wide range of values, there are no changes in the rates of increase, it is quite clear that the rate of increases is not conforming to the logistic law of development.

When, moreover, one is dealing with human populations, there are reactions to the inherent impulse to increase which depend on ideas arriving from outside the region considered, for example, those which affect the standard of living and the whole economic situation. Again the powers of a community are such that all limitations of their immediate environments are affected by the skill with which they can satisfy their general wants either directly or by exchanges with outside peoples. And further, economic and ethical considerations may lead to the intensification or the diminution of the impulse to increase, apart from any continuity in the physical, internal, or external environments, or in their economic conditions. Moreover economic considerations have an intimate relationship with the social ideals of the people. Probably the whole matter can be illustrated best by reference to the growth of the population of the United States of America, to which we shall now turn, in order to see how far its growth may properly be regarded as conformable to the logistic curve.

8. Tests as to the legitimacy of fitting a logistic curve to population results. The question "Do populations actually conform to the logistic curve?" has been raised by Professor Raymond Pearl, Dr. L. J. Reed, Mr. G. Udny Yule, and others, the last named having shown on the one logistic curve, drawn to the same "standard time-scale," the populations of the United States (1790–1920), of England and Wales (1801–1921), and of France including Alsace-Lorraine (1801–1911).[1] If they could justly be regarded as an appropriate totality combination, this would seem to confirm the logistic character of populations. That they cannot properly be so regarded will appear later.

First of all it is to be noted that *some part of a logistic curve can readily be drawn through many series of results,* in such a way that individual deviations shall be a minimum, forasmuch as part of the curve is concave upwards, near the point of inflexion it is sensibly linear, and the last part of the curve is convex upwards. Individual results will lie on either side of any curve taken to represent them generally; on the whole they do not afford a justification of the logistic view of change of rate, unless they generally conform to the consequences of the assumption.

Of all populations which might be expected to furnish a test, that of the United States stands in a unique position. In 1790 it was only 3.929 millions and in 1920 it had attained to as great a value as 105.711 millions. The physical environment of the population is one of very great natural wealth. The density has grown steadily from about one-fiftieth of the supposed maximum population according to the logistic theory, to over one-half thereof. As a matter of fact, however, the population values for successive decades indicate rates of increase very close to a common value of 0.346 from 1790 to 1860, then a sudden change to lesser but also approximately uniform ratios of increase of 0.239 from 1860 to 1910. This will now be established.

9. Population Curve of the United States. In the curve studies of his *Human Biology* Professor Pearl has deduced the upper limit of the United States population as 197.274 millions. In the work hereunder we shall take 200 millions as a round figure to test the applicability of the theory. The actual populations represented as fractions of the possible total will then be as shown in column (IV) in Table I.

The respective areas and population numbers between 1790 and 1920 are as shown in columns (II) and (III) in the Table. If the population had increased at a uniform rate from 1790 so as to attain to what it actually was in 1860, it would have been as set forth in column (V). This average ratio increase would have been from 100.000 to 134.596

[1] See *Journal of the Royal Statistical Society*, January, 1925, pp. 1–62.

per decade, an increase per decade of 34.596 per cent, or from 100.000 to 103.0156 per annum, that is to say 3.0156 per cent per annum.

TABLE I

DECENNIAL POPULATIONS OF THE UNITED STATES, 1790–1920

(I) Year	(II) Square miles (1000)	(III) Actual population (1000)	(IV) Population fraction	(V) Population at constant annual rate (1000)	(VI) Population at constant decennial rate (1000)	(VII) Actual increase per cent
1790	868	3,929	.0196	3,929	3,924	
1800	868	5,308	.0265	5,288	5,281	35.103
1810	1,686	7,240	.0382	7,118	7,108	36.383
1820	1,754	9,638	.0482	9,581	9,567	33.130
1830	1,754	12,866	.0643	12,895	12,877	33.486
1840	1,754	17,069	.0853	17,357	17,332	32.671
1850	2,944	23,192	.1160	23,361	23,328	35.868
1860	2,974	31,443	.1572	31,443	31,398	35.579
1870	2,974	38,558	.1928	42,224*	.....	22.629
1880	2,974	50,156	.2508	56,963*	.....	30.077
1890	2,974	62,948	.3147	76,670*	.....	25.504
1900	2,974	75,995	.3800	103,194*	.....	20.727
1910	2,974	91,972	.4599	138,895*	.....	21.025
1920	2,974	105,711	.5286	186,949*	.....	14.938

Had the annual rate continued to 1910 the figures marked with an asterisk would have been attained, that is, it would have been 186.9 millions approximately in 1920, a population it has not even yet attained (at the end of 1925). Retaining the ratio 1.34596 per decade the figures would have been somewhat closer to the actual, if the value for 1790 was 3923.5 or say 3924; see column (VI). Column (VII) shows the actual increases per cent per decennium.

In equation (3) if we write

$$(7) \quad q=r(1-y/L)$$

q will be the inherent rate of increase as reduced by the fact that the remaining population is only $L-y$, that is, relatively it is $(L-y)/L$ instead of unity. The factors of r are therefore unity diminished by these numbers, i.e., they are 0.9804, 0.9735, etc. From these and the first three results in column (VII), it would appear that the inherent rate per cent per decennium would be about 35.93, 37.15, 34.63, or say about 35.90 per cent per decennium. We shall take it as 36 per cent

which is the same as 3.1226 per cent per annum, or an instantaneous rate of 3.074847 per cent; that is, the value of r is .03074847. Thus the rates at the years in question are these multiplied by the remaining relative possible-population factors, and expressed decennially are the values of q in Table II.

TABLE II

RELATIVE POSSIBLE-POPULATIONS, AND RATES FOR THE UNITED STATES

r_0=0.0307487=36.00 per cent per decade

Year	Rel. poss. pop.	q	Actual q	Year	Rel. poss. pop.	q	Actual q	Year	Rel. poss. pop.	q	Actual q
1790....	.9804	.030146		1840...	.9147	.028126		1890...	.6853	.021072	
			.030017				.030651				.018836
1800....	.9735	.029934		1850...	.8840	.027182		1900...	.6200	.019064	
			.031030				.030438				.019082
1810....	.9618	.029574		1860...	.8428	.025915		1910...	.5401	.016607	
			.028578				.020352				.013922
1820....	.9518	.029267		1870...	.8072	.024820		1920...	.4714	.014495	
			.028884				.026296				
1830....	.9357	.028772		1880...	.7492	.023037					
			.028271				.022717				

Assuming that in each decennium the rates of increase were uniform during the decennium, their instantaneous values would be as shown in the final columns.

It was immediately evident from column (VII), Table I, that the rates of increase did not conform to the logistic curve. In Table II it will suffice if we take for comparison the means of the rates at the initial and final dates of the several decennia. Expressing these per 10,000 and referring them to the central date they are:

Year	1795	1805	1815	1825	1835	1845	1855	1865	1875	1885	1895	1905	1915
A....	300	310	286	289	283	307	304	204	263	227	188	191	139
B....	300	298	294	290	284	277	265	254	239	220	201	178	156

On line A are the actual rates, and on B those which, very approximately, would result were the population numbers consistent with the logistic theory. It is very evident from the progressions of the actual rates that they cannot be taken as representing logistic results.

10. What governed the rates of increase in the United States? The factors governing the rates of increase have already been referred to in Section 3 herein. The population of 1860 is given from that of 1790 by a ratio of 1.35496 per decennium; that of 1910 from that of 1860

by 1.23955, or of 1920 by 1.22396 per decennium. The rate from 1810 to 1820 was greater instead of being less than that of 1790 to 1800. Again it was increasing from 1810 to 1830 and, though falling slightly from 1830 to 1840, increased again from 1840 to 1850. This larger rate remained nearly constant till 1860. All this is inconsistent with the ordinary logistic curve. The decline of the rate from 1860 to 1870 was without doubt due to the Civil War. This discloses the fact that the fitting of a logistic curve is quite *an empirical proceeding.*

It may be noted here that F. A. Walker,[1] the American statistician and economist, pointed out many years ago (1888) that in 1790 there were about 600,000 white families, few very rich, few very poor. Food was abundant, social traditions and religious beliefs encouraged fecundity, domestic tranquillity prevailed. The land was but partially settled. Up to 1850 instead of mechanical labor, farm laborers, at least in the north, mainly consisted of young men who worked for a few years in order to acquire a little ready money, so as to marry. Such conditions allowed population to expand without restriction. But between 1840 and 1850 a change came, notwithstanding enormous accessions of peasant blood from Europe, increments from Ireland and Germany being extremely large. (See "Immigrants" columns, Table III.) The migrants bred with unprecedented rapidity. Nevertheless the high rate of increase was greatly reduced. This was due to the population passing from simplicity of life to comparative luxury. *The rise in the standard of living*, the multiplication of artificial "necessities," the extension of paid domestic service, the introduction of women into factory labor, were factors in limiting the power of increase. Then from 1861 to 1865 came the war of secession, with an estimated loss of 1,765,000 persons up to the 1870 figures for population. The census of 1880 was said to have confirmed the view that social and economic factors had operated as the causes of the reduction of the rate of increase.

The view of F. A. Walker is virtually that the social outlook of a people, which changes the standard of living and also the degree of reproductivity of a people, as well as their fundamental attitude to reproduction, must necessarily be reflected in the rates of increase. That changes of that character occurred, appears among other things in the relative increases of the urban part of the population. From 1790 to about 1831 the percentage of urban population grew much more slowly than it did from 1831 to 1880. Thus the increases were as follows:

(8) 1790 to 1831, $3.40+0.07t_1$; 1831 to 1880, $6.34+0.28t_2$

[1] See *Encyclopædia Britannica*, Ninth Edition, XXIII, p. 818.

in which formulae t_1 is the number of years after 1790 and t_2 the number after 1831. Thus the change from 1831 to 1880 was no less than four times as great as the change from 1790 to 1831, and this was due to changes of the people in their industrial character. A similar phenomenon has appeared in Australia, and these facts show that in the mode of increase we have to do with such causes; in short that the elementary assumptions which lead to the logistic curve have at best only a very modified application. It was that there were new developments in the social-economic life of the population of the United States which altered its power to increase and which have prevented its ever again attaining the original rate.

By attending to the nature of the case, it is readily seen that this inhibition of the original rate of increase is not that assumed in the fundamental equation of the logistic theory, viz., that increase is more difficult because the relative population density has become greater. There can, for example, be no doubt in the mind of any student of national economy that, given a thrift and energy equal to that characteristic of the period 1790 to 1850, and standard of living as humble, the rate would be much greater than it is.

11. Is the population of France consistent with the logistic theory? The population of France for every five years from January, 1801, onward was about as follows, the populations having been obtained by linear interpolations, and expressed in millions:

Year.....	1801	1806	1811	1816	1821	1836	1841	1846	1851	1856
Population.	27.35	29.11	29.35	30.00	32.42	33.46	34.18	35.30	35.76	36.02
Difference..	1.76	.24	.65	2.42		.72	1.12	.46	.26	1.34

Year.....	1861	1866	1876	1881	1886	1891	1895	1901	1906	1911
Population.	37.36	37.41	35.73	37.53	38.17	38.34	38.51	38.93	39.23	39.59
Difference..	.05		1.80	.64	.17	.17	.42	.30	.36	

The numbers for 1826, 1831, and 1871 are omitted, and those from 1876 onward exclude Alsace-Lorraine. The results, though somewhat irregular, show that the growth is not so great for the period 1876 to 1911 as it was from 1801 to 1866, and also that in this last mentioned period the later increase was on the whole less than it was primarily. Thus a logistic curve can be made to fit the results very well, as has been shown by Mr. G. Udny Yule.[1]

[1] *Journal of the Royal Statistical Society*, January, 1925, p. 21. See, however, Professor A. L. Bowley's results on p. 78.

Moreover, it may be noted, France's population is an *old* one, and its density is 184 per square mile, while that of the United States is a *new* one with a density of only 35 per square mile. Thus the part of the logistic curve representing France may be expected to be later than the point of inflexion of the curve, while that to represent America will be before the point of inflexion is reached. This condition is also satisfied. When, however, all the facts are considered, it is seen that the elements of increase from 1886 to 1911 do not support the logistic theory. The successive quinquennial percentages of increase are 0.44, 0.44, 1.09, 0.77, 0.92, which generally *are increasing instead of diminishing*, and reveal the fact that the fitting of the logistic curve is but empirical. A review of the percentage of the increases from 1801 to 1821 and from 1836 to 1861, leads to the same conclusion. It may be added that with results so irregular, the period of 110 years is an insufficient basis to be taken as a support of the logistic theory.

Owing to the absences of proper censuses the data unfortunately are not available for a very definitive examination of the theory, but there are other countries which throw some light upon it, and we shall later consider these.

12. Population increase with a fixed area. Before embarking on the more general examination of the increase of populations it is proper to note that the laws of increase may be complicated not only by migration factors, but also by change of the area under review. For the United States the area changes have been shown in Table I, Section 9. Restricted to the area as at 1790 the populations are presented in Table III, and have the decennial percentages of increase varying greatly. They do not show the diminishing rates required by the logistic curve: for example for 1810–1820 we have the rate 22.3 per cent, and for 1870–1880 the rate 21.5 per cent, the population having

TABLE III

POPULATION GROWTH OF THE UNITED STATES IN THE 1790 AREA

Year	Population (millions)	10 year increase (per cent)	Immigrants	Year	Population (millions)	10 year increase (per cent)	Immigrants
1790.....	3.929		....	1850.....	14.569		1.713
		33.5				18.9	
1800.....	5.247		....	1860.....	17.326		2.598
		29.1				13.6	
1810.....	6.779		....	1870.....	19.688		2.314
		22.3				21.5	
1820.....	8.293		....	1880.....	23.926		2.812
		23.5				17.8	
1830.....	10.240		.143	1890.....	28.188		5.246
		15.0				19.0	
1840.....	11.781		.599	1900.....	33.534	....	3.844
		23.7					
				1910.....		....	7.754

increased from about 7.5 to about 21.8 millions. (It should have been about 20.6 millions.) The great irregularities show that the diminishing population possibilities are not the main factor. It should be noted here that so long as an area allows of a population expansion, equally as freely as at the commencement of the occupation, the rate of growth may even increase. (See Australian results given hereinafter.)

13. Supposed conditions of the geometric growth of a population. Suppose a population of *uniform character* freely developed in a territory also of uniform character and of relatively unlimited extent; then *ex hypothesi* each element of its increase would have exactly the same opportunity and the same impulse to grow. In such a case the growth would be almost *exactly* geometric, as was fully recognized by Verhulst. Where a territory is relatively limited, however, the increase may not always have the same environmental opportunity as its predecessors; hence the density of population may be, and ultimately certainly will be, one of the factors involving a decrease of rate. But changes in the character of the people, or in their environment, will also cause great changes in the rate of increase.

The notion that the *growth may be geometric up to a limit* when the population is *normal* (Verhulst), and then change abruptly into a logistic population is not, of course, an impossible one, and if the tangent of the two curves were common at the point of change, the assumption would be intrinsically satisfactory, and quite appropriate for examination. It is also possible that, initially, an increasing density may so accelerate the facilities for occupation as to *increase the rate of growth:* this appears in America. Compare 1790 to 1800 with 1800 to 1810. Australia's population does not afford a perfect means of testing this, because of the effect of the "gold rushes," which tended to increase the population abnormally. The results of successive decades are, however, given in Table IV, the decennial ratios being shown in the last line.

TABLE IV

POPULATION OF AUSTRALIA IN MILLIONS AND ITS INCREASES

Year.....	1790	1800	1810	1820	1830	1840	1850	1860	1870	1880	1890	1900	1910	1920
Population..	.0021	.0052	.0116	.0335	.0700	.190	.405	1.146	1.648	2.232	3.151	3.765	4.425	5.411
Ratio.....		2.48	2.23	2.89	2.09	2.71	2.13	2.83	1.44	1.35	1.41	1.20	1.18	1.22

Australia being a *new* country with a density of only a little over 1 per square mile, it can hardly be regarded as subject to a limitation of

the possibility of increase owing to an appreciable approach to its population limit L. Its variations of increase are therefore significant as showing that the conception that increase depends on the ratio P/L, has very little to support it, at least, where this ratio is small, and also that the ratio can even then vary between relatively somewhat wide limits. It thus appears that there have been periods when the rate of increase was accelerated, for example in the periods 1870 to 1885, and 1900 to 1915 (see Table V). Presumably this was due to favorable conditions somewhat of the character hereinbefore suggested.

TABLE V

QUINQUENNIAL RATIOS OF INCREASE IN AUSTRALIA

Year	1870	1875	1880	1885	1890	1895	1900	1905	1910	1915	1920
Population	1.648	1.898	2.232	2.695	3.151	3.492	3.765	4.033	4.425	4.970	5.411
Ratio		1.15	1.18	1.21	1.17	1.11	1.08	1.07	1.10	1.12	1.09

Arithmetical Average 1.128. Quinquennial ratio, over all, 1.12626.

The same thing is seen in the populations of England and Wales between 1851 and 1861. Mr. Yule has shown [1] that their census populations fit a logistic curve very closely. Adopting the censuses of 1801, 1851, and 1901, he obtains the limiting population 91,211 millions, $1/r = 61.134$ years, the point of inflexion 1936.065, and for the individual results of the other censuses the figures shown in Table VI.

TABLE VI

POPULATIONS OF ENGLAND AND WALES, 1801 TO 1911, (MILLIONS)

Year	1801	1811	1821	1831	1841	1851	1861	1871	1881	1891	1901	1911
Actual	8.89	10.16	12.00	13.90	15.91	17.93	20.07	22.71	25.97	29.00	32.53	36.07
Logistic	8.89	10.29	11.88	13.68	15.69	17.93	20.40	23.11	26.05	29.19	32.53	36.03
Difference		+.13	−.12	−.22	−.22		+.33	+.33	+.08	+.19		−.04
Parabola, Line		10.14	11.94	13.85	15.87	18.00	20.24	22.59	25.93	29.27	32.61	35.95
Differences		−.02	−.06	−.05	−.04	+.07	+.17	−.12	−.04	+.27	+.08	−.12

But the results are more closely represented from 1811 to 1871 by a *parabola*, $10.14+1.745\ T_1+0.055\ T_1^2$, and then for the next four decades by a *straight line* $22.59+3.34\ T_2$, the times T_1 and T_2 being expressed in decennia. In view of the fact that the population of England and Wales is also an *old* one, it was not to be expected that it would *not*

[1] *Journal of the Royal Statistical Society*, January, 1925, p. 8.

have reached its inflexion stage, though the change from the parabola to a straight line is consistent therewith. One is faced however with the fact that the logistic curve is not really the most accurate representation of the general progression of the population, in short the logistic curve has merely been empirically fitted to the data.

14. The secular growth of populations. Based upon the counts of the Tung-hwa-Luh, the populations of China in millions from 1715 to 1885 were approximately as shown on line (I) of Table VII, while those of several countries in the western world, collectively, which had accurate statistics for 85 years later were as shown on line (II), after they had been multiplied by 0.9859 to make the 1715 to 1835 aggregates equal. These countries are Australia, Austria, Belgium, Denmark, Finland, France, Germany, Greece, Hungary, Italy, Netherlands, Norway, Portugal, Servia, Spain, Sweden, the United Kingdom, and the United States.

TABLE VII

POPULATIONS OF CHINA AND CERTAIN OCCIDENTAL COUNTRIES, 1715–1895

(Line (II) has an 85 year lag.)

Year	1715	1735	1755	1775	1795	1815	1835	1855	1875	1895
(I)	133	154	187	235	288	348	403	430	435	436
(II)	135	160	193	229	276	342	413	...	...	...
Difference	+2	+6	+6	−6	−12	−6	+10	...	...	...
(III)	133	154	187	232	289					
Difference	0	0	0	−3	+1					

The general similarity is very striking, but it is important to notice that the differences of increase over successive 20 year periods are also significant. In the Chinese populations they reach a maximum and then fall off, but in the western world they continue to increase.

TWENTY YEAR DIFFERENCES

China (I)	21	33	48	53	60	55	27	5	1
Occidental countries (II)	25	33	36	47	66	71	..	..	..

The increases in (II) however are so irregular that, solely from the numbers themselves, no conjecture can be made as to the probable future increases. From general considerations, however, it appears certain that they are not likely to approach a limiting value like those of China within say a century. The reason for this is that the technical and economic condition of the western countries is such that appreciable growth is still possible.

The results from 1715 to 1795, as calculated by the parabola $133+15T+6T^2$, in which the unit of T is 20 years, are as shown on line (III) in Table VII; the parabola well represents the results to 1795 for either population but gives excessive values beyond it. The results after 1795, however, cannot be regarded as very reliable, but are in any case quite inconsistent with the continuation of the parabola. Neither will the ordinary logistic curve fit them at all well.

The actual population aggregates for the countries in the western world mentioned above are set forth in Table VIII.

TABLE VIII

POPULATION OF A GROUP OF OCCIDENTAL COUNTRIES, 1800–1920

Year	1800	1820	1840	1860	1880	1900	1920
Population	137	162	196	232	280	347	419
Calculated	137	165.25	196	233	280	340.75	419
Difference	0	+3.25	0	+1.0	0	−6.25	0

The third degree curve, $137+56.5T-2.5T^2+5T^3$, gives the figures on the line "Calculated," showing how closely it represents the results. A logistic curve will not fit them as well.

As a test, the value of an aggregate of populations of countries with accurate statistics depends upon the fact that minor oscillatory effects are likely to cancel each other, excepting that, insofar as they are due to world conditions, they are presumably common to all. Thus, if the hypothesis leading to the logistic curve is a valid one, it may be expected that such an aggregate will closely conform thereto. The fundamental characteristic of the logistic population growth is that the rate of increase is always diminishing as the population increases. We shall therefore consider the rates of increase of these aggregates.

The rates of increase (see Table IX) are somewhat irregular, but are non-logistic; in percentages they are as shown on line (I) and, if represented as on the whole increasing linearly, would be as set forth on line (II).

TABLE IX

PERCENTAGES OF INCREASE IN A GROUP OF POPULATIONS

(Populations in Millions)

	1800–1820	1820–1840	1840–1860	1860–1880	1880–1900	1900–1920	
(I) Actual	18.2	21.0	18.4	20.7	23.9	20.8	
(II) By linear assumption	18.3	19.2	20.1	21.0	21.9	22.8	
Differences	+0.1	−1.8	+1.7	+0.3	−2.0	+2.0	

	1800	1820	1840	1860	1880	1900	1920
Population by (II)	136.6	161.6	192.6	231.3	279.9	341.2	419.0
Actual Population	137	162	196	232	280	347	419
Differences	−0.4	−0.4	−3.4	−0.7	−0.1	+5.8	0

The populations by (II), calculated backward from 419.0, show differences from the actual populations which are less than those previously given by the third degree curve. Thus it is seen that the hypothesis of rates linearly increasing represents the facts better than any other simple one, and the populations do *not* at all conform to the logistic conception.

For Australia the quinquennial percentages of increase from 1860 to 1920 slowly diminish, sensibly linearly, as presented in Table X.

TABLE X

QUINQUENNIAL INCREASES OF AUSTRALIAN POPULATIONS, 1860 TO 1920

Actual	1.15	1.18	1.15	1.18	1.21	1.17	1.11	1.08	1.07	1.10	1.12	10.9
Linear	1.21	1.20	1.19	1.18	1.17	1.16	1.15	1.14	1.13	1.12	1.11	1.10
Differences	+.06	+.02	+.04	0	−.04	−.01	+.04	+.06	+.06	+.02	−.01	+.01

If the populations be representable by the logistic curve, the absurd result is reached that the point of inflexion was antecedent to 1900 and the limit of population would be under 6 millions, although 6 millions would be only about 2 persons per square mile. The quinquennial *percentage* of increase from 1870 to 1920 was 12.626, from 1860 it was 13.812. These results show that the logistic curve is not substantiated by the Australian population, though that population must be deemed free to expand, as far as possible food supplies, etc., are concerned. In short it lends no support to the logistic hypothesis.

15. Fitting power of the logistic curve not proof of the theory. The form of the logistic curve may be so altered by variations of r and L, that portions of it may be made to fit sensibly (a) linearly increasing coordinates, (b) a curve of increasing values either convex downwards or (c) one convex upwards, or again (d) one consisting of a curve consisting of (b), (a), and (c) tangentially arranged, provided that a point can be so taken on the straight portion that curves (b) and (c) are sensibly symmetrically situated in respect thereto. Thus it may empirically be fitted to very many population curves. Such fittings, however, do not afford any proof that the *hypothesis* from which the curve is deduced is necessarily correct; they merely disclose the fact that a curve of the type in question, in which both the vertical and

horizontal scales, viz., that of L and that of t, can be altered as one pleases, and any part of the whole range of the curve can be used, has great "fitting" possibilities. In other words, the fact that part of the logistic curve can roughly express the successive numbers of a population, does not *per se* establish its validity, nor justify the deductions as to the limiting values of the population.

A general consideration of all the data compels the view that, for limited periods, populations tend, under definite and constant conditions, to grow at a constant rate, so long as such conditions last; and also that changes of rate merely reflect the environmental advantages or disadvantages and alterations in the intrinsic characters of the population. Actually the tendency to increase is always subject to fluctuations, though doubtless it is also true that ultimately it must be subject to a progressive decrease depending upon progressive environmental and other limitations.

(*To be concluded*)

THE HARVARD BUSINESS INDEXES—A NEW INTERPRETATION

By Karl G. Karsten

I

It is the task of this article to show that the valuable A, B, and C Curves of the Harvard University Committee on Economic Research are not, as hitherto there seemed good ground for supposing, true "lags" of each other. I shall try to show that they do not foretell each other, as has been assumed, in the order of speculation, business, and money. Yet I find that they do bear an extremely simple relation to each other, and that the interpretation hitherto placed upon them, as mutually self-predicting indexes, might well yield to a different interpretation, one which has enabled me during the past two years successfully to forecast the course of these curves six months in advance of their publication.

As everybody knows, the Harvard Committee has rendered a lasting service to the business world in recording by means of these curves, which cover the past twenty years, the general movement of conditions in the commodity, security, and money markets. In point of technique these indexes stand alone among business index numbers. A common unit of measurement, the standard deviation from trend, has been used to give equal amplitudes to all their component elements. They have also a high value since the elements have been carefully chosen, each representing a group of economic series with more or less simultaneous and similar movements. Lastly, they have certain constant interrelations, by means of which business men and investors may gain a look into the future. It is in the interpretation of these interrelations, on which depends the value of the curves for forecasting, that the pioneer work of the Harvard Committee can be improved upon.

The A Curve is called an index of speculation conditions, because it is a composite of different series reflecting conditions in the New York Stock Exchange. Originally, it comprised New York bank clearings, railroad stock prices, industrial stock prices, and railroad bond yields for the pre-war period. In the post-war period it was reduced to an average of the New York bank clearings and the industrial stock prices. In 1923, monthly data for the pre-war period were published, in which the A Curve consisted of New York bank clearings, industrial stock prices, and shares traded. All of these different combinations produce approximately the same index numbers with only minor

variations, because the several series joined have similar curves. They may be described as combinations of price and quantity statistics relating to speculation.

The B Curve is called an index of business conditions, and has had a much more varied history as to its composition. It originally contained the records of pig-iron production, bank clearings outside New York City, New York Bank Reserves, Bradstreet's prices, and Bureau of Labor Statistics prices. In 1924, it was republished without the statistics of the New York Bank Reserves. For the post-war period it be-

TABLE I

A BANK ACCOUNT ("A") AND ITS NET CHANGES ("B")

	Monthly			Total Account "A"
	Deposits	Checks	Change "B"	
January	340	−300	40	40
February	400	−280	120	160
March	520	−310	210	370
April	130	−250	−120	250
May	850	−370	480	730
June	500	−410	90	820
July	215	−115	100	920
August	215	−145	70	990
September	130	−410	−280	710
October	30	−300	−270	440
November	30	−280	−250	190
December	330	−250	80	270
January	310	−260	50	320
February	420	−320	100	420
March	620	−280	340	760
April	340	−250	90	850
May	250	−210	40	890
June	140	−260	−120	770
July	45	−285	−240	530
August	25	−255	−230	300

came a combination of bank clearings outside New York, and the ratio between an index made from ten sensitive commodity prices and Bradstreet's prices. This last feature has been omitted in 1926, and the B index becomes a simple index of bank clearings outside New York and Bradstreet's prices for the pre-war period, and of bank debits outside of New York City and the United States Bureau of Labor Statistics prices for the post-war period. Thus the B index is also a combination of price and volume or value statistics. The various changes in the constitution of the index have made only minor differences in the actual shapes of the curves, because the elements were chosen, in the first place, for their similarity of movement.

The C Curve is described as an index of money or banking conditions, and like the other curves is a composite of several series. For the pre-war period it originally comprised New York bank deposits, New York bank loans, and the interest rate upon 60–90 day paper, and upon 4–6

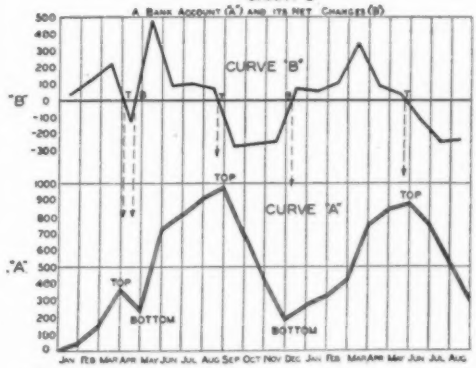
months paper. In later revisions of the curve for the pre-war period, the group was reduced to an average between the two interest rates on 4–6 months paper and on 60–90 day paper only. For the post-war period it was reduced to the ratio between 60–90 day commercial paper rates and railroad bond yields. It has been revised in 1926 very slightly, but the C Curve remains an index only of statistics relating to the price of money, and all the various forms of it have the same shape with differences which are negligible.

These three index number series are fully corrected for seasonal variations and for trends. Trends have been fitted to each of the component series by the Harvard Committee, and revised from time to time as experience shows them to have been wrongly extended into the future. The indexes are, furthermore, all simple averages of the various series which they comprise, after the fluctuations of each series about its trend, corrected for seasonal variations, have been reduced to a uniform amplitude. In this last respect the Committee has taken an important step in the technique of using time series.

The Harvard Committee could not be expected to note that these three curves belong to a certain category of peculiar paired-curves, because such a category was not generally recognized in economics when the Committee formulated its methods.[1] Yet the failure to notice this relationship introduces a large element of uncertainty into the Harvard forecasts. Before going on to demonstrate the true forecasting relation of the curves, I wish to make clear how this true relation has been obscured. It is hoped that the method here proposed will appeal to the Committee, and will be adopted by it in continuing its extremely valuable work.

The interrelation of the curves is that made familiar to all in a personal bank account, of which an example is presented herewith. The monthly bank balance and the net changes in it produced by depositing and drawing checks are plotted in Chart I, and the figures are shown in Table I.

CHART I



This chart shows the curves of a series of data (Curve B) and of the derived series (Curve A) formed by cumulating the items of the first series.

[1] This category of paired-curves was first described in economics and business statistics by the present writer in this JOURNAL, March, 1924.

Of course, when a check is drawn it has an immediate effect upon the balance. There is an undoubted causal relation between the two curves. The heights of one curve correspond with the changes in the heights of the other; the values of the one comprise the successive differences or changes in the values of the other. Yet the two curves represent entirely unlike mathematical operations; the second curve portrays a "cumulative," or series of sub-totals to date, of the values of the first curve. They must therefore differ, since the curve of the net changes in one's bank account can hardly be the same as the curve of the bank account itself.

Yet it will be noted that these two curves seem to resemble each other. The cumulative curve seems to parallel the net-change curve after a "time-lag" or delay of one or two or more months.

But while such superficial resemblance is apt to appear between paired-curves of this class—which are technically known as integrand and integral functions of finite variables—the resemblance usually does not prove either regular or reliable. It may at any time entirely cease. While the resemblance holds, of course, the one curve seems to parallel the other in advance, but predictions based on this seeming parallelism would be hazardous.

Indeed, so long as the descriptions attached to the curves clearly tell their peculiar interrelation, these curves would not be likely to deceive anyone. But when no such easy explanation is attached, even the expert at curve analysis might, unless especially skilled in detecting this type of relation, be deceived into mistaking this false resemblance for a true parallelism, in which one curve foreshadows the other. He might even try to establish the false relationship by "lagging" one curve, so that it is superposed over the other with a plausible coefficient of correlation. Thus the direct and nearly simultaneous relationship between the two might escape discovery.

Mathematics, physics, and—we are just beginning to learn this—economics, also, present many cases of this class. Often the false or superficial resemblance between the two components of the pair, after one of them has been "lagged," is quite impressive while it lasts. But the true relation in this class of paired-curves is not revealed by lagging. It consists in the fact that as the one component crosses its normal or trend line, the other component has at that moment a top or a bottom, a peak or a valley, a maximum or a minimum (call it what you wish).

The relation between the two curves is definitely causal, the first either recording the cause of the second or recording some force which varies with the cause of the second. In mathematics, when the curves

are sine curves, the second is a trace of the first, paralleling it after the lag or interval of one quarter-cycle. Hence the relation is said to be in quadrature. And so long as the curves continue to resemble sine curves it would, of course, be possible to predict the second from the first. But when the resemblance to sine curves is perhaps fortuitous and certainly undependable, it is not safe to base predictions upon the hope of a continued parallelism.

In business and economics, the curves are not regular and have no constant cycle period, so that the lags representing a quarter-cycle phase are irregular and not dependable. It is a dangerous proceeding to study the vague long time-lag between the top of one curve and the top of the other—the point is to think of the simultaneous interaction or coaction, *whereby one curve has a turning-point the very moment that the other crosses its zero or normal line.*

This is true of the curves of the bank account. Other examples of these paired-curves may be found of a similar nature. For example, business men are regularly dealing with figures showing the net warehouse receipts-or-withdrawals and cumulating these to get the figures of stocks on hand. And again in economics, this is the typical relation between "flow" or "stream" statistics and "fund" or "stock" data. But another familiar illustration, that of a reservoir, will be readily grasped by everyone.

Let it be a reservoir then, with a combined intake-and-outlet pipe. Suppose that we keep a record of the flow into or out of the reservoir, or lake, and also keep a record of the heights of the water-level in the lake at the same times. We might plot a curve of the flow in the pipe, representing inflow by positive values drawn above the zero line, and outflow by negative figures plotted below the zero-line, while we would draw the time of no-flow upon the zero-line itself. We also might plot a curve of the heights of the water at these times. We might label the two curves, Curve B and Curve A.

Now the height of the water-level in the reservoir varies, not with the flow of water through the pipe, but with the cumulative of that flow. Thus when the inflow through the pipe is at its fastest, the level in the reservoir would not be at its highest, for the reservoir would go on rising as long as any inflow continued, no matter how small. Instead, the high-water level would occur when the inflow ceased, just before the outflow began.

In other words, the top of Curve A would *coincide exactly in point of time with the crossing of normal* by Curve B. This is the quadrature or cumulative relation.

Incidentally, there might be a lagged resemblance or parallelism in

these curves for quite a while, but it would be fortuitous and incidental. It might suddenly cease and thus betray any confidence reposed in it.[1] But the cumulative relation would continue, and it may be relied upon.

This phenomenon, *the peak and valley of one curve appearing simultaneously with the intersection of the zero-line of the other*, as in the case of the reservoir curves, is invariable in all cases where the cumulative relation of cause and effect exists.

In our examples we have named the first curve A and the second curve B, since in their essentials they correspond with the Harvard A and B curves. Every such pair of curves may possess the deceptive characteristic shown in our examples, of seeming to parallel each other for a longer or shorter interval after a time-lag.

Perhaps it escaped the attention of the Harvard Committee that the curve of the level of prices in the stock market, Harvard Curve A, has a top or bottom, a high or low-water mark, whenever the curve of the flow of money into or out of the stock market from or into the business world—represented by Harvard Curve B, the curve of business conditions—crosses its normal trend or zero line. If they did not note this fact, the Committee would readily fall into the natural trap which such paired-curves afford, during the uncertain period in which they are paralleling each other. That is, the Committee would naturally center its attention upon the fact that a top in one curve has preceded the top in the other, and that the bottom of one has preceded the bottom in the other.

The time-lag, computed on this basis, as the Committee has frequently pointed out, varies considerably; that is, between four and ten months. Despite this broad variation the Committee has given weight to the apparent sequence of events, and has made it the basis of a system of forecasting. In that system the object, after one of the curves reaches a top or bottom, is to foretell that the other will do likewise sometime in the next four, five, six, seven, eight, nine, or ten months—one never knows which beforehand![2]

[1] Only if the reservoir were connected with the sea, so that the tide flowed in and out through the pipe, then of course the highest water-level would occur regularly three hours after the greatest flow in the pipe, and the lowest level would be three hours after the greatest outflow. But it would be wrong to assume therefore that the water-level is directly corresponding to the flow in the pipe after a three-hour lag, for a heavy storm might keep up the inflow many hours and the high water mark would then be many hours behind the fastest tidal race.

[2] To quote the Committee, "Our interpretation of the fluctuations of the three curves of the current index chart is based upon the relationship which was discovered among similar curves for the period 1903–14. All three curves had wave movements, but the upward swings and the crests of the waves were not reached simultaneously. The major movements of speculation preceded those of business by 4 to 10 months, and the movements of business preceded those of banking by 2 to 8 months." "Monthly Survey of General Business Conditions," in the December, 1921, issue of the *Review of Economic Statistics*, Preliminary Volume 3, Number 12, page 371.

II

Not only has the Committee's attention been diverted in this way to a fortuitous and undependable time-lag or lagged parallelism, between its A and B curves—which, as we see, move one after the other only while their types continue to be alike—but the Committee has furthermore mistaken *which of the two really moves first.*

Thus the Committee has the A Curve forecasting the B Curve, while as we have observed in all such examples, the B Curve is really the causal curve and the A Curve, its cumulative, is the resultant or effectual curve.

How came the Committee to put the cart before the horse? For the very simple reason that the B Curve is plotted, so far as its relation to the A Curve is concerned, just upside down. This makes the valleys of the B Curve appear as peaks and, vice versa, the peaks as valleys.

Now the true peak of the B Curve must precede the peak of the A Curve, if the one curve is the cumulative of the other, and the true valley of the B Curve then follows behind. But if we plot the B Curve upside down, that true B valley following the A peak will appear as a B peak following the A peak. And so through all the other turns, the upside-down B peaks and valleys follow in point of time the A peaks and valleys. Plotting in the wrong way readily betrays one into interpreting the B Curve as following the A Curve; plotting in the right way shows that it precedes.

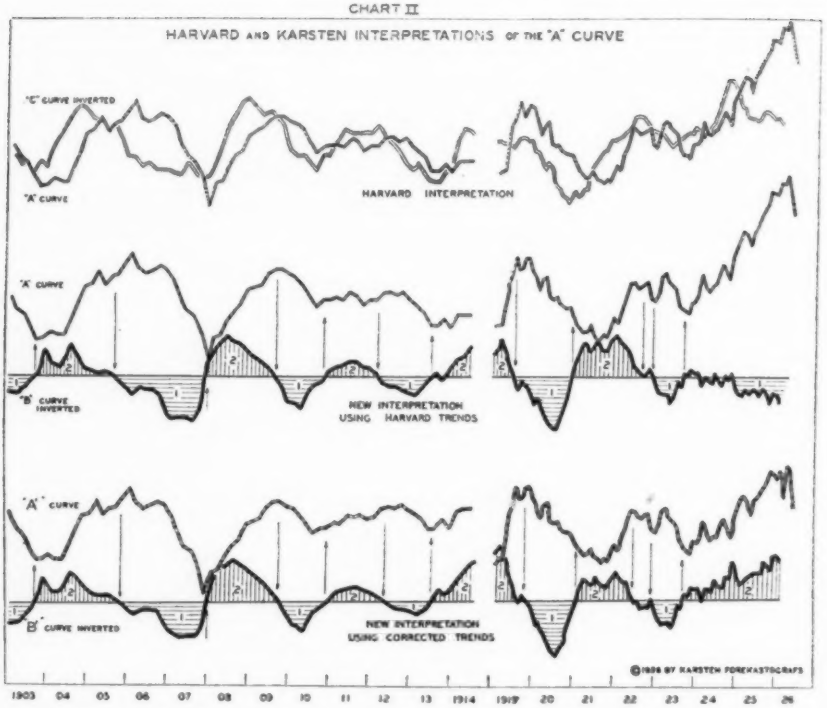
To substantiate these audacious statements, we have two proofs. One is a logical proof, in which we shall try to show reasons why one or the other interpretation is correct. The other is a statistical proof, in which we measure a similarity of behavior by a "coefficient of correlation," and so learn which theory yields the greater degree of accuracy in its results.

The logical proof is very simple. The relation between business, the price-level in the security markets, and the flow of money into or out of those markets is quite clearly one in which the price-level is the result, or cumulative, of the flow of money. In other words, the price-level varies with the stock of money in the market.

This, indeed, is an elementary theorem in economics, that the general level of prices in any market mainly varies with the quantity of money in that market; the price itself being merely the ratio between the goods sold and the money given in exchange for them.

There are other factors in the full equation of exchange, such as the number of transactions and the velocity of turnover of the money. But the most important factor in applying this equation to the stock market, since the supply of securities varies only slowly with the issu-

CHART II


HARVARD AND KARSTEN INTERPRETATIONS OF THE "A" CURVE
"C" CURVE INVERTED
"A" CURVE
HARVARD INTERPRETATION
"A" CURVE
"B" CURVE INVERTED
NEW INTERPRETATION USING HARVARD TRENDS
"A" CURVE
"B" CURVE INVERTED
NEW INTERPRETATION USING CORRECTED TRENDS
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1903 04 05 06 07 08 09 10 11 12 13 1914 1919 20 21 22 23 24 25 26

According to the Harvard interpretation, the A Curve of Speculation is predicted about four months ahead, by the C Curve of Money Conditions, inverted.

According to the new interpretation, stock market prices are controlled by the flow of money, which is evoked by business conditions according as they are good or bad, and shown by the B Curve of Business, inverted. Periods of business prosperity are shown by the areas marked (1), depressing the stock market by drawing money out of the market into the more profitable channels of business. Periods of business depression are shown by areas marked (2) and exert a bullish influence on the stock market by pouring money into investments from business as it grows less profitable, thus inflating security prices. Hence the stock market turns whenever one or the other of these periods terminates.

The coefficients in support of this theory are very high (.94 and .96), as compared with the Harvard coefficients (.58 and .72), respectively, shown on Chart III.

The curves on this chart have been plotted to represent data compiled by me according to the Harvard Committee formulae for these curves.

ance of new securities, is the money available for it. Hence the *supply of money is, broadly speaking, the cumulative of the flow of money* into or out of the investment market.

But the flow of money into or out of the investment market is, of course, a direct function of business conditions.

For money can go only into one of two places, if it is to earn a return

or yield without leaving the country; that is, into business projects or into loans and securities and other investments. We must either use our funds ourselves or rent them out to others who will use them.

When business conditions are such as to make profits attractive, it follows that we use all the funds ourselves which we can employ in our business. To this end we sell securities, if necessary, or borrow on them, or borrow from a bank which will call in loans to investors. Thus the money is drawn into business, coming, as it must, out of the investment market. And when business conditions are worse than normal and do not promise attractive profits, we reduce our output. We seek to operate on a small supply of working capital and reinvest our spare funds. Or we leave them at the bank, which will have to reinvest them. Or we merely do not make fresh loans at the bank, so that the bank must lend to others who will invest, or must invest itself.

It is easy to visualize this flow of the margin of available funds into or out of the investment market as business conditions get better or worse. The point is that so far as the stock market is concerned poor business conditions, since they set up a flow of money into the market, represent a positive force; good business conditions, on the other hand, since they pull money out of the market, represent a negative force.

Thus poor business conditions should be plotted on the chart upward from the normal line and good conditions should be plotted downward from the normal line. Since this is just the reverse of what has been done by the Harvard Committee, we conclude that the Harvard B Curve, so far as its relation to Curve A is concerned, has been plotted upside down.

True, regarded solely as a picture of business conditions, Curve B should be plotted exactly as the Harvard Committee has plotted it; it would be confusing if presented otherwise. But when it is used in making an analysis of the true relation between general business and the securities market, the business curve must be inverted.

The logic of the situation follows inexorably. If we dare to think of Curve A as the cause of Curve B, then we arrive at the conclusion that the stock market predetermines, governs, controls, and makes or unmakes the business conditions of the nation. This is a length to which even the most myopic Wall Street inhabitant would hardly dream of going, good as the idea may be for I. W. W. propagandists or agrarian politicians!

If we do not go so far as this, and merely say that Curve A foreshadows Curve B, then we are confronted with what would be one of the

miracles of modern economics, for we surely would be at a loss to explain by what occult powers the speculating world in mass could accomplish a thing which no speculator seems able to do individually, and that is correctly to divine and discount the future of business, for between six months and a year in advance.

Investors do discount in advance the dividends to be distributed by business concerns, but that is invariably done upon the basis of the latest reports of earnings and profits and the general present nature of business conditions out of which those future dividends will be paid. In short, the investor watches present business reports to discount future net worth of business concerns and hence security prices thereof. But "present business reports" are sometimes six months old or out of date and always at least a few weeks or even a month or two old; and all the business so-called "barometers"—which are nothing but business records in the aggregate, such as carloadings, bank clearings, unfilled orders, and the like—are similarly from one to several weeks out of date.

This means that business reports as of the recent past are really the basis of investors' discounting of the future. These reports measure conditions that were in the making or were operating even earlier, for a report covering a period of time is issued at the end of the time. So we see clearly that it is business conditions of the past that determine the individual investor in his judgment of the future. It is hard to see how the mass psychology of all investors taken together can be foreshadowing the future, when every one of the individual investors is reacting to the past.

III

The statistical proof, as we have said, consists in measuring the degree of similarity or movement of two curves after allowing for the "proper time-lag." The measure used is the so-called "correlation coefficient," which reveals the degree of correspondence by its nearness to 1.00.

The Harvard Committee has stated that "the maximum correlation between the pairs of curves is fairly high; and this fact, taken in connection with the lags between B and A and between C and B, establishes the validity of the revised index for forecasting business conditions on the basis of the sequence of movements in these curves."[1] Again, "According to these coefficients, B lags six months after A; and C lags four months after B, and ten months after A."[2] And

[1] *Review of Economic Statistics*, January, 1924, p. 17.
[2] *Ibid.*

again, "A decline of Curve C usually precedes, and hence forecasts, an upward movement of Curve A," [1] and "a sharp decline in Curve C began, as has been said, some ten or twelve months previous to a substantial rise in Curve B" [2]—and, therefore, we may conclude, some four to six months previous to a "substantial rise" in Curve A.

The coefficients of correlation thus invoked by the Committee are as follows: Curve A when lagged six months correlated with Curve B with a coefficient of .77 during the pre-war period (in the post-war period I find the coefficient to be only .73); and Curve B when lagged four months correlated with Curve C with a coefficient of .80 during the pre-war period (in the post-war period I find that it is only .36). As to Curve C when lagged four months and inverted, I find that it correlated with Curve A with a coefficient of .58 during the pre-war period and of .72 during the post-war period. In short, so far as the Harvard theory of the relation between Curves A and B is concerned, the statistical proof consists of coefficients of correlation reaching only .77 and .73 before and after the war, respectively. Moreover the Committee's explanation of the A Curve, that it follows the inverted and lagged C Curve, is ill-supported by the coefficients .58 and .72 before and after the war, respectively. The Committee's explanation of the C Curve we shall take up later. It is true that many statisticians have viewed coefficients of .50 or more with complacency. But more exacting standards can and should be set in the study of economic relationships.

Certainly the Harvard coefficients are not high enough to constitute a proof of the Harvard hypothesis concerning the relations between the movements of Curves A and B. Karl Pearson, who largely developed the coefficient of correlation, issued a warning in regard to the interpretation of a coefficient that was less than .60, and in my own forecasting work I do not consider of much value a coefficient below .90; in fact, I prefer to work with coefficients of .95, or better.

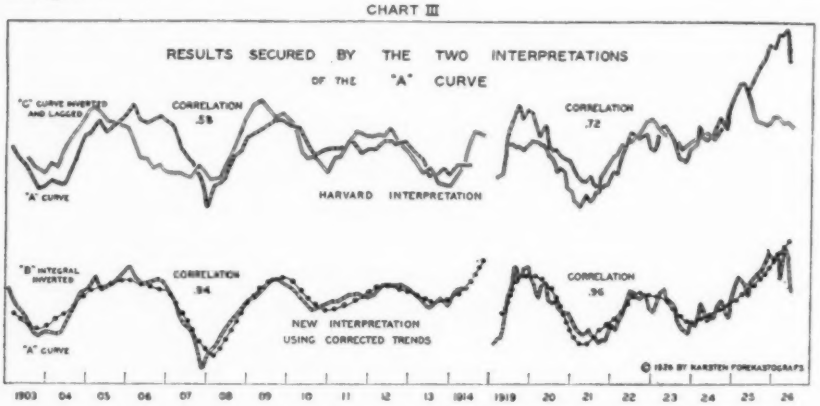
I have already sketched out the logic of the "cumulative" interpretation of these curves, whereby Curve A, the price-level of securities, shows the cumulative effects of the flow of money (into the market out of business, or out of the market into business), whereby these prices are inflated or deflated. It remains to make a statistical test of this hypothesis, using for this purpose the measure proposed by the Harvard Committee.

To do this we need merely take the data of the Harvard B Curve, as published by the Committee, and cumulate the figures of the deviations

[1] *Ibid.*, December, 1921, p. 371.
[2] Warren M. Persons, *Interpretation of the Index of General Business Conditions*, 1922, p. 10.

from normal (i. e. correct or true trend), thus obtaining a statistical series which represents the cumulated effects of the B Curve. The cumulating is done just as the net change figures of column B in Table I are cumulated to get the total bank balance to date, shown in column A. The cumulated series can be compared with the Harvard A Curve. If, as the quadrature theory demands, the Harvard A Curve really represents the cumulated effects of the B Curve, then there must be a close correlation between the Harvard A Curve and the cumulation of the Harvard B Curve—we may call it the Harvard B cumulation, or integral.

CHART III


RESULTS SECURED BY THE TWO INTERPRETATIONS
OF THE "A" CURVE
"C" CURVE INVERTED AND LAGGED
"A" CURVE
CORRELATION .58
HARVARD INTERPRETATION
CORRELATION .72
"B" INTEGRAL INVERTED
"A" CURVE
CORRELATION .94
NEW INTERPRETATION USING CORRECTED TRENDS
CORRELATION .96
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1903 04 05 06 07 08 09 10 11 12 13 1914 1919 20 21 22 23 24 25 26

The Harvard interpretation is shown by plotting the C Curve of Money Conditions inverted, and lagged four months, so as to conform to the Harvard theory, according to which the C Curve so treated predicts the A Curve by that period of time.

In accordance with the new interpretation the curve of the cumulative (or integral) of the Harvard B Curve of Business is shown to predict the Harvard A Curve by three months, with greater precision than the Harvard interpretation permits. This interpretation is supported by the very high coefficients (.94 and .96), as compared with the Harvard coefficients (.58 and .72), respectively.

The curves on this chart have been plotted to represent data compiled by me according to the Harvard Committee formulae for these curves.

The new coefficient of correlation works out .94, as compared with the Harvard coefficients of .77 and .58, before the war. After the war the new coefficient is .96, as compared with .73 and .72. These coefficients are extremely high for general statistics of monthly data, and it is a high tribute to the Harvard indexes that, all unknown to their authors, they behave in such a significant way.

Thus we have a mathematical measure justifying our belief that the cumulative of the Harvard B Curve comes much nearer to paralleling the Harvard A Curve with only a slight lag (of two or three months) than does the B Curve itself, with any lag in either direction you please.

I may add a word about the risk that is run in the attempt to interpret apparently long time-lags between causes and effects in the business world—in this day of the telegraph, the newspaper, the radio, and the aeroplane, not to mention the railroad and the automobile, and the volume of current business statistics and the sensitiveness of the credit structure. I have no objection to distributing the effects of a business condition over one, two, three, or four months following, and I can see that in some ways, such as through long-time purchasing, their effects may be felt over even longer periods.

But though these long drawn-out effects may be a part of the situation, they must be a very small part of it, and the chief effects must take place within a very few weeks, or perhaps even days, after the event. This is the era of hand-to-mouth buying. Just so it is a period of hand-to-mouth forecasting over short intervals of time. For the time elapsing between a causal force and a reacting or effectual phenomenon, in the modern business world, is not likely, in the main, to be more than two or three months. Although others may not agree with this view, a lag of half a year or eight or ten months between true cause and true effect seems to me an absurdity, to which no one, no matter how in love with statistical methods, should allow his statistical evidence to carry him.[1]

IV

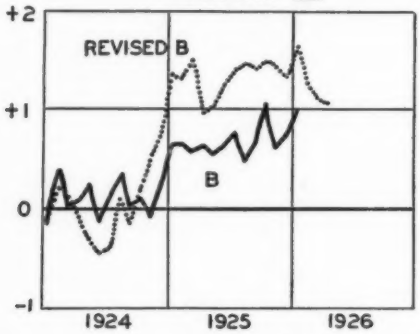
It is now necessary to bring up an entirely different aspect of the case, and, taking the Committee's occasional revisions of the trends of its indexes, to show that the latest revision in which, by changing its trend, the Committee raised the level of the B Curve during 1925 and 1926, may not be well considered. My view is that the B Curve should have been revised in the opposite direction, by taking quite another trend correction out of it.

I think that the Committee is quite correct in reserving the right to revise the adjusted trend-lines of its index numbers from time to time, and so altering the level of its curves. For trend-lines are really only ruler-edges laid across curves so as, roughly speaking, to bisect the curves and leave equal areas between the curves above and below the trend-lines. Trend-lines are intended to describe some imaginary state of "normalcy" wherewith to compare the actual conditions depicted by the curve. It is obvious that at best we can only approxi-

[1] In this it should be understood that I am considering the major economic and business forces, such as the time between the demand for money during a business boom, and the resultant stiffening of the interest rates; or the time between the flow of selling orders into the market because of the greater attractiveness of investing one's marginal funds in one's own business, and the resulting "bear" movement of deflation of security prices.

mate such trends in the past, and that we cannot project such lines reliably very far into the future. We must be ready from time to time to revise our estimates of what the true "normal conditions" were. But while the Committee's method of analysis leaves it perforce groping for this normal trend, the quadrature method of analysis affords a clear and simple proof of the location of the normal line of the B Curve, that is, the net-change curve.

CHART IV



The Harvard Committee has recently revised the B Curve trend and so shifted the B Curve to a higher level. It will be seen from the other charts that this was in error, as the B Curve should have been lowered.

The curves on this chart have been plotted to represent data compiled by me according to the Harvard Committee formulae for these curves.

I believe that no one would be more ready to admit that these trends and estimates of normal are inconclusive than the Harvard Committee itself. Indeed, the Committee has explicitly stated that its estimates need revision, and has on two or three occasions revised its indexes explicitly for the purpose of more correctly tracing the normal trend.

It is now possible by the cumulative method to have for the first time a definite, intelligent indication of the normal trend. For the normal is shown by the level of the B Curve at the moment when the A Curve changes its direction; that is to say, the normal line for any causative force is such that its cumulative curve has a top or bottom at that moment. It follows from this that whenever a peak or valley occurs in the cumulative the causative force is crossing normal, and is thereby showing us what its normal condition is; hence a trend-line drawn through such points would trace the true course of normal.

This may be explained simply. If I tell you just when my bank balance was greatest you know at what moment I stopped putting money into and began drawing money out of the bank—that is, you

know just when the flow of money was neither positive or negative, but was actually zero. And from this you could plot the zero-point on the curve of that flow of money if you had that curve before you properly drawn but not calibrated. In short, you could at such moments locate the zero (or normal) point on the B Curve.

This illustrates a general feature of the quadrature or cumulative relationship which is of enough importance to warrant explaining it clearly. What really happens is as follows: In the first place we cumulate the deviations of the B-type curve from whatever trend or normal line we think best suited for the B Curve. Second, we compare this cumulative with the A-type curve (after changing both to common amplitudes, such as standard deviations). If parallelism appears we may suspect a cause-and-effect relationship, if this seems logical. In any event what we must really work with is the residuals—that is, the differences between the B cumulative and the A Curve (in common amplitudes). If these residuals are constant, they tell us that we have merely cumulated from the wrong level (that is, used the wrong "constant of integration"). If the residuals are linear, we know that they would disappear after changing the level of the original trend or normal line of the B Curve. If the residuals fit a parabolic curve, we know that they would disappear if a change be made in the original B trend, that change corresponding to a parabola of one less degree. So long as this does not impose upon us an illogical or unreasonable trend, we may use the quadrature method of analysis not only to establish the quadrature relation between two curves, but also to correct the trend already fitted to one of them.

In Chart II this correction has been made. The arrows connect the critical points on the two curves. The lowest pair of curves shows them as corrected for trends which, through this method of analysis, as we shall later see in discussing the C Curve, are indicated for each curve.

The Harvard Committee has passed arbitrary trend-lines through the curves of the original data, so that the B Curve takes its level above or below the normal or zero line entirely on account of, and with reference to, those trends.

Judging entirely from the relations which the foregoing analysis has disclosed, we should say that the Harvard B Curve has been too high and should be lowered below its zero line, as far as A is concerned, so that in our chart where this curve is inverted, it will be above the zero line and represent a bullish influence, accounting for the great rise in the stock market.

Such an analysis discloses the fact that business conditions have been in this particular respect, at least, considerably below normal during

the last two years, and when we recall the great amount of gold in this country and the fact that there has been no inflation, such as was threatened, we may perhaps feel that conditions have been much less prosperous than would have been normal in view of our abundant supply of gold. It may be true that we have not been conscious of a great depression, for it may have been that the normal line was really rapidly rising during this period, so that curves of business conditions which seemed to be moving sidewise were really relatively falling below that normal line. Certainly, so far as the flow of money into or out of the stock market is concerned, we know that there has been a great inflow, and this is evidence of the flow of money away from the business channels which would have seemed to be the natural goal.

We are therefore confronted today with two possible ways of viewing the Harvard B Curve. It may be merely a good index of the flow of money into or out of the stock market, as suggested above; or, it may be in addition a significant index of business conditions as is claimed by the Harvard Committee, and in that sense indicate that business has been subnormal; although we have not yet fully recognized this condition. If the latter interpretation is correct, then we may expect a real business boom to begin in the not distant future, which will be of surprising magnitude to the business world. In any case, if the past behavior of the B Curve means anything, it means that the B Curve will in time rise so far that the Harvard Committee will find it necessary to reduce its level by adopting a decidedly different trend.

Without respect to whether the B Curve is an index of business conditions of one kind or of all kinds, or merely a statistical function and predicting device for the A Curve, it has shown this "cumulative" relation with the A Curve. On the basis of this fact, making use of the A Curve to determine the trend of the B Curve, I suggest that the B Curve should have been very much lowered, instead of being raised, as in the latest Harvard revision. In making the suggestion I am taking the A Curve as a criterion for the B Curve normal line. Of course, if the A Curve itself is seriously at fault, then some further correction of the B Curve will be necessary.

In the accompanying charts (Charts II and V) the trends of the A and B Curves have been corrected only where so stated, and then only in the following way. The A Curve has been corrected by the C Curve and therefore is correct only if the C Curve trend is correct, and the B Curve trend has been corrected by the A Curve and is correct only if the A Curve trend is correct.

It will be seen that the correction of the trends used by the Harvard Committee is in no way essential to proving the correct interpretation

of the curves, but rather that such interpretation makes possible the correction of trends. This correction of trends can take place frequently, as each new turn of the cumulative or A Curve is a fresh signal of the whereabouts of the normal (or trend) line of the integrand or B Curve, because at that moment the B Curve must be crossing normal. I have not done more in the present writing than to apply very simple and obvious corrections of the trends used by the Committee.

V

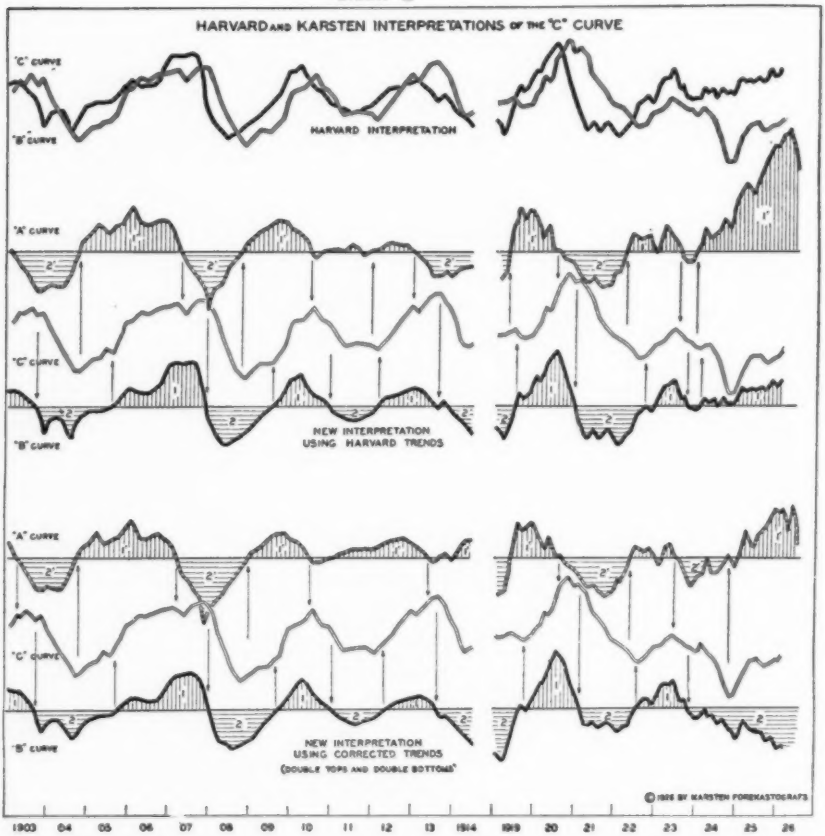
The discussion of the C Curve will be brief. Logically we may call it the price-of-money curve, as it consists of interest rates. Now the price of money is the resultant of the demand for money, cumulatively to that demand. Thus so long as the demand for money is greater than normal the price continues to rise and does not cease rising until the demand falls below normal. Again it is well to think of the water-level in the reservoir. Only, in the case of money, business conditions must be regarded as a positive force when better than normal, and as a negative one when worse than normal. That is to say, with respect to the C Curve, the B Curve should not be inverted. And the A Curve, showing market conditions, also affects the C Curve.

In short, the C Curve is an interesting example of a cumulative of two causal forces; it therefore tends to show double tops and double bottoms, the first of which is the result of the B Curve crossing the trend-line, while the second is made by the A Curve crossing the normal line. Logically, the C Curve is to a great extent the cumulative of both the A and B Curves, for the price of money is largely determined by the demand for money either for business purposes or for investment purposes, that is, either in the business world or in the security markets. It is noticeable that the business force seems to be the stronger and more influential of the two.

Statistical proof of this hypothesis also is available. The Harvard Committee, as we have seen, rests its interpretation upon a correlation of .80. To test our interpretation, we have only to cumulate the data of both the B and the A Curves, and having so reached cumulative series which show net effects, combine the two by averaging (with the B Curve cumulative weighted more heavily, say three times as heavily). Thus we get a single combined series of figures to which the Harvard C Curve should, if this theory be correct, closely correspond.

This has been done and the degree of correspondence is shown by the correlation coefficients of .85 and .94, which contrast favorably with the Harvard coefficients of .80 and .36.

CHART V



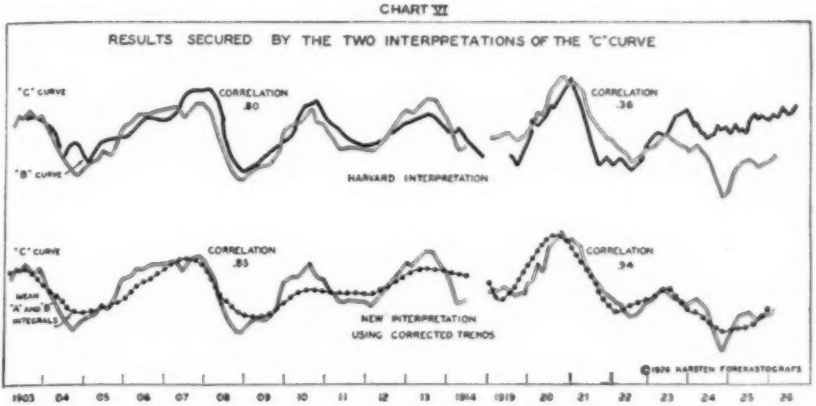
According to the Harvard interpretation, the C Curve of Money Conditions is predicted by the B Curve, about six months in advance.

According to the new interpretation the C Curve of Money is the result of the interchanging good and bad conditions shown by the shaded areas (1) and (2) in the A Curve of Speculation and B Curve of Business. The first period of prosperity is shown by the areas marked (1) in the B Business Curve and (1^1) in the A Speculation Curve; and throughout these periods money rates are stiffening. The periods of depression are shown by areas (2) in the B Business Curve and (2^1) in the A Speculation Curve; during these periods money rates are softening. It will be seen that money rates turn promptly as either of the curves crosses the zero line, that is, whenever one area is terminated and another area is begun.

In support of this theory the coefficients of correlation (.85 and .94) are higher than the Harvard correlation coefficients (.80 and .36), respectively.

The curves on this chart have been plotted to represent data compiled by me according to the Harvard Committee formulae for these curves.

CHART VI


RESULTS SECURED BY THE TWO INTERPRETATIONS OF THE "C" CURVE
"C" CURVE
"B" CURVE
CORRELATION .80
HARVARD INTERPRETATION
CORRELATION .36
"C" CURVE
MEAN "A" AND "B" INTEGRALS
CORRELATION .85
NEW INTERPRETATION USING CORRECTED TRENDS
CORRELATION .94
©1926 HARTFEN FORECASTOGRAPH
1903 04 05 06 07 08 09 10 11 12 13 1914 1919 20 21 22 23 24 25 26

The Harvard interpretation is shown by plotting the B Curve of Business Conditions lagged six months, so as to conform to the Harvard theory, according to which it predicts the C Curve of Money Conditions by that interval of time.

In accordance with the new interpretation, the curve of the average between the cumulatives (or integrals) of the Harvard B Curve of Business and the Harvard A Curve of Speculation is shown to predict the Harvard C Curve with greater precision than the Harvard interpretation permits. This single predicting curve is an average or composite of the two predicting cumulatives.

Properly two predicting curves instead of one would be used alternately for these correlations, to correspond with the double tops and double bottoms of the C Curve of Money Conditions; but for the sake of simplicity of presentation a single average predicting curve is here used, with the result that the correlations for the two periods are not quite so high as the two predicting curves would indicate.

This interpretation is supported by correlation coefficients (.85 and .94), which are higher than the Harvard coefficients (.80 and .36), respectively.

The curves on this chart have been plotted to represent data compiled by me according to the Harvard Committee formulae for these curves.

VI

In the foregoing we have set forth a theory of the relations between the three Harvard curves, and shown, statistically, that it seems to fit the facts better than the theory advanced by the Harvard Committee. We have shown that Curve A is the cumulative of Curve B, and that Curve C is the cumulative of the two curves A and B taken together. This theory treats Curve B as the generating force, of the three, and treats the other two curves as depending upon and deriving from the changes in the business curve.

Our theory does not explain the movements of Curve B itself, but merely the movements of Curves A and C. To explain Curve B we must have recourse to other business statistics. In another article I hope to show how Curve B, the course of business conditions, can be and is

explained, and even predicted, by the use of a few business "barometers" and the cumulative method here described.

The theory upon which this new interpretation is based was originated by Mr. Charles N. Edge, and was first described by the present writer in this JOURNAL in March, 1924. The original theory of cumulative relations in economics was independently developed, in part, by Professor Irving Fisher and Colonel Ayres. The writer has been indebted also to Mr. Carl Snyder, Professor Wesley C. Mitchell, Dr. Frederick R. Macaulay, and Professor William F. Ogburn, for the lively interest and valuable advice which they have given toward the development of this theory and the technique of its application.

The quadrature method of analysis has been applied to many economic movements, particularly the prediction of the course of the price-level of common stocks on the New York Stock Exchange and of the Bond price-level. It also seems to explain the movement of interest rates and of the wage level. In all probability it will yield, also, a means of predicting the general wholesale price-level and even the prices of the more important commodities. Its application to the three Harvard curves was first observed by me in March, 1924, and was then discussed confidentially with Colonel Ayres and with Mr. Snyder, and has been watched for two years before announcing it.

REQUIREMENTS FOR STATISTICIANS AND THEIR TRAINING [1]

I

STATISTICAL TEACHING IN AMERICAN COLLEGES AND UNIVERSITIES [2]

By James W. Glover, *University of Michigan*

The Committee on Educational and Professional Standards sent out in 1925 to American Colleges and Universities a questionnaire on instruction in statistics, requesting name of course, prerequisite in mathematics, content of course, hours per week divided as to lecture and laboratory, semester or term, graduate or undergraduate, and instructor's name and rank. The information was requested for the college year ending in June, 1925, separately with respect to various departments in which courses in statistics are usually given.

The sub-divisions were as follows:

I. Statistical and allied courses given in the Department of Mathematics.
(Statistical method, probabilities, least squares, actuarial)
II. Statistical courses in Departments of Economics and Social Science.
III. Statistical courses in Schools of Business.
(Business statistics, business cycles, managerial statistics, report analysis)
IV. Statistical courses in other Departments.
(Education, biology, public health and medicine, psychology, astronomy, extension, etc.)

General questions called for statements as to mathematical or statistical laboratories, statistical libraries, and calculating and tabulating machine equipment. The questionnaire schedule was rather long and detailed, and a complete return required, perhaps, more work than might fairly be expected of those to whom it was sent. However, about 125 institutions reported with replies in such form that most of them could be included in this report. Tables I, II, III, and IV appearing in the body of this report are drawn from the schedules submitted.

[1] A summary of certain papers read at two sessions of the Eighty-seventh Annual Meeting of the American Statistical Association, New York City, December, 1925.

[2] The Committee is much indebted to Mr. Edwin W. Kopf for assistance in preparing and sending out the questionnaire, and in tabulating the returns.

In order to eliminate detail, courses reported by only one or two institutions are not included; this omission in no way affects the general conclusions one would draw from a report including all these details. Courses in actuarial mathematics are not included in this report.

TABLE I

STATISTICAL COURSES IN DEPARTMENTS OF MATHEMATICS

Colleges reporting	Elementary statistics	Advanced statistics	Theory of probability	Least squares
Courses in	*57	40	21	14
No courses in	65	93	97	97
Prerequisite in mathematics				
None	5	1		
High School	8	1		
†College Algebra	19	5	4	1
‡Freshman	14	12	4	5
Calculus	11	21	13	8
Hours per week				
Lecture or recitation				
1–3	49	34	17	9
4–5	8	6	4	5
Laboratory				
None	51	33	20	14
1–3	5	7	1	
4–6	1			
Period of instruction				
One semester	41	28	16	13
Two semesters	16	12	5	1

* As a matter of fact this figure (57) is the *number of courses* offered, while (65) is the *number of colleges* reported as not giving courses.

† This is one-half year of college mathematics and may include trigonometry.

‡ This usually includes college algebra, trigonometry, and analytic geometry.

This table indicates that the departments of mathematics in 57 out of 122 colleges reporting were offering courses in elementary statistics in the college year 1924–25, with most of them requiring one-half year to one year of college mathematics and 11 of them the calculus also.

TABLE II

STATISTICAL COURSES IN DEPARTMENTS OF ECONOMICS AND SOCIAL SCIENCE

Colleges reporting	Elementary statistics	Advanced statistics	Business cycles	Business forecasting
Courses in	86	8	4	3
No courses in	56	103	107	108
Prerequisite in mathematics				
None	46	4	3	2
High School	7	1		
College algebra	23	1		1
Freshman	10	2	1	
Hours per week				
Lecture or recitation				
1–3	78	8	4	3
4–6	8			
Laboratory				
None	45	3	4	1
1–3	36	4		2
4–11	5	1		
Period of instruction				
One semester	53	4	2	3
Two semesters	33	4	2	

Advanced courses in statistics were offered in 40 of these institutions, of which 21 required the calculus. The three hour course through one semester predominates, but only about 6 colleges have adequate laboratory equipment. Courses in the theory of probability and least squares are very common in addition to those in mathematical statistics.

In Table II, statistical courses in the departments of economics and social science, 86 out of 142 colleges report courses in elementary statistics. Of those offering courses, 46 require no mathematical preparation; 7, high school mathematics; and the remainder, one-half to one year of freshman mathematics. Two and three hour courses for one term and two terms are about equally divided. Laboratory work is more general than in departments of mathematics, as about half of the institutions report laboratory work as required. Only 8 colleges report advanced courses in statistics and 4 of these make no requirements for mathematics. Only 7 report courses in business cycles and business forecasting and 5 of these require no mathematical preparation.

TABLE III

STATISTICAL COURSES IN SCHOOLS OF BUSINESS

Colleges reporting	Elementary statistics	Advanced statistics	Business forecasting	Business cycles
Courses in	53	3	7	8
No courses in	73	109	107	106
Prerequisite in mathematics				
None	21		5	4
College algebra	12	2	1	3
Freshman	19	1	1	1
Calculus	1			
Hours per week				
Lecture or recitation				
1–3	45	2	6	8
4–7	8	1	1	
Laboratory				
None	15		5	5
1–3	28	2	2	3
4–8	10	1		
Period of instruction				
One semester	33	2	5	6
Two semesters	20	1	2	2

In Table III, statistical courses in schools of business, 53 out of 126 report courses in elementary statistics, of which 21 require no mathematical preparation and the rest require one or two semesters of freshman mathematics. The courses are about equally divided as to two or three hours per week and about half of them require one to three hours per week of laboratory work; 20 of these courses are continuous throughout the year. A course in business forecasting is offered in 7 and business cycles in 8 institutions. No mathematics is required in 9 of these institutions and one-half to one year of freshman mathematics is required for the remainder. No calculus is required in the advanced statistics offered by 3 colleges.

TABLE IV

STATISTICAL COURSES IN DEPARTMENT OF EDUCATION

Colleges reporting	Elementary statistics	Advanced statistics	Educational surveys	Educational measurements
Courses in	34	10	4	24
No courses in	89	107	111	100
Prerequisite in mathematics				
None	17	5	4	15
High School	3	1		
College algebra	7	1		8
Freshman	7	2		
Calculus		1		1
Hours per week				
Lecture or recitation				
1–3	27	6	3	20
4–6	7	4	1	4
Laboratory				
None	25	8	4	19
1–3	5	1		5
4–10	4	1		
Period of instruction				
One semester	24	8		14
Two semesters	10	2	4	10

In Table IV, statistical courses in other departments, education, vital statistics, psychology, and agriculture are included. In the department of education 34 colleges out of 123 report courses in elementary statistics; 10 out of 117, courses in advanced statistics; and 24 out of 124, courses in educational measurements. More than half these courses require no mathematical preparation and most of the remainder only high school mathematics or one semester of college algebra. In one instance only, calculus is required for advanced statistics. A surprisingly small number of departments of education offering courses in statistics require laboratory work.

TABLE IV (*continued*)

STATISTICAL COURSES IN DEPARTMENTS OF PUBLIC HEALTH, PSYCHOLOGY AND AGRICULTURE

Colleges reporting	Public Health		Psychology	Agriculture
	Elementary vital statistics	Advanced vital statistics	Elementary statistics	Elementary statistics
Courses in	11	3	12	4
No courses in	104	109	104	108
Prerequisite in mathematics				
None	10	3	9	1
High School	1		3	
College algebra				2
Freshman				1
Hours per week				
Lecture or recitation				
1–3	8	2	11	4
4–6	3	1	1	
Laboratory				
None	5	1	4	
1–3	3		7	4
4–9	3	2	1	
Period of instruction				
One semester	9	1	10	4
Two semesters	2	2	2	

Eleven departments of public health in 112 colleges offer elementary courses in vital statistics, and three offer advanced courses in vital statistics. In all but one case no prerequisite in mathematics is required. Laboratory work is required, as a rule, and 4 colleges report 2 semesters of instruction. In departments of psychology 12 colleges out of 116 offer elementary courses in statistics, mostly with no mathematical prerequisites and 4 of these departments require no laboratory work. In agriculture 4 out of 112 colleges provide an elementary course with from one-half to one year of freshman mathematics and one to three hours of laboratory work.

In the questionnaire it is brought out that 56 out of 115 colleges report having calculating machines. This in most cases means one or two adding machines, a hand operated calculating machine such as the Monroe, Marchant, Brunsviga, etc., and a few slide rules and drawing sets. Only a few reported extensive equipment running into as many as ten calculating machines. Only 4 institutions have tabulating and sorting machines such as the Hollerith and Powers. With regard to auxiliary devices 30 colleges report from one to twenty-five slide rules; 22 colleges, from one to twenty-five drawing tables; and 15 colleges, from one to thirty drawing sets. Only 8 report planimeters, 3 report integraphs, and 3 report having harmonic analysers.

With regard to library equipment the report is quite unsatisfactory, probably because it required too much work to get the information. Out of 113 colleges reporting, 28 have statistical libraries. As a rule, however, it may be said that the books and journals on statistics are scattered through all the various departments of each college and university and it would be quite hopeless to determine very closely the total library equipment in statistics.

It is difficult to draw conclusions from these tables, although a general tendency is perhaps evident. The percentage of institutions giving elementary and advanced courses in the four major departments offering courses is as follows:

	Department			
	Mathematics	Economics	Business administration	Education
Elementary statistics	47%	61%	42%	28%
Advanced statistics	30%	7%	3%	9%

This indicates roughly that the highest percentage, 61 per cent, of elementary courses is offered in economics, mathematics following with

47 per cent. In advanced statistics, as might be expected, mathematics leads all the others, as 30 per cent of the reported institutions give these courses, which naturally require a preparation beyond the calculus. It is also evident that the mathematical prerequisites in departments of mathematics, even in elementary statistics, are much stiffer than those of the other three departments. On the other hand it will be noted that the departments of mathematics require less hours of work in the laboratories than the other departments.

Perhaps, after all, the net result of this report is to confirm the judgment based on observation that departments of mathematics as a rule require freshman mathematics as a minimum prerequisite for elementary statistics and are satisfied with nothing short of a thorough training in the differential and integral calculus for advanced statistics, while other departments are content with no mathematics beyond high school algebra for elementary statistics and in many cases not much more than this for their courses in advanced statistics. This divergence in point of view and practice will probably continue for a long time and the employer looking for trained men—not experts—must judge for himself from which department he will select his assistants. Only time and experience will settle questions of this character and perhaps the results accomplished by the men who are trained along these different lines will eventually decide the matter.

II

STANDARDS AND REQUIREMENTS IN STATISTICS

WHAT IS EXPECTED OF STATISTICIANS BY EMPLOYERS IN THE FIELD OF BUSINESS

By Donald R. Belcher

A contributor to the *Bankers Magazine of London*, contrasting the status of business research in Great Britain with that in this country, recently said, "In the United States life is conducted from the cradle to the grave as a statistical experiment, man's works and all God's creations are made material for statistical exercises." Statistical exercises connected with the phenomena of birth and death, as well as those of health and education in the interim, can best be presented by others. As to "man's works and all God's creations," there are some statistical aspects in which the business organization is concerned, and I wish to discuss the personnel standards and requirements for successful progress in this field.

Without doubt my personal views are of necessity somewhat colored

by the characteristics of the business with which I happen to be associated. The American Telephone and Telegraph Company, as you know, is the parent organization of the Bell Telephone System and has for one of its principal functions the maintenance of a general staff for the benefit of the 25 Associated Operating Bell Telephone Companies. Within this system there are inter-connected today over 16,000,000 telephones, any one of which can be put into direct communication with any other one anywhere in the United States. You, as a telephone subscriber, are familiar with your telephone instrument, with the telephone poles and wire upon the highway, and, in less tangible form, with the "voice with a smile" at the central office that handles your call. You probably have little conception of the vast amount of hidden property—switchboards, underground conduits and cables—which is also required to furnish the service of communication to those 16,000,000 telephones. Yet the telephone plant in active service in the Bell System today has cost nearly $3,000,000,000, and some $350,000,000 of similar plant is being constructed each year for additions and replacements. The construction of telephone plant on so great a scale, however, is only a preliminary step in the furnishing of telephone service. The furnishing of service cannot be conducted on a wholesale basis. It requires every 24 hours the making of some 50,000,000 telephone connections, each of which represents a separate individual transaction between the telephone company and the public which it serves.

I recite these figures merely to make clear that the Bell System, by virtue of its size, the geographical distribution of its activities, and the technical complexity of its operation, provides what is probably an ideal field for the application of statistical research. Somewhere within its confines there must be represented practically every form of statistical research, varying all the way from simple enumeration up through the most complex applications of mathematical and statistical theory. The problems of supply and price trend of raw materials; manufacturing costs; distributing costs; markets and market structures; labor supply; wages; living costs; interest rates; the future trend of wages, prices, and interest rates; the cyclical ebb and flow of present and future general business conditions—all of these problems require statistical approach and the rigorous application of statistical technique. All of these problems and many others must be analyzed by the telephone organization, not only because they are essential elements in the conduct of its own operations, but also because they provide an index of the future economic conditions of the country as a whole and therefore forecast the magnitude of future demand for its services.

So large and so varied, then, is the application of statistical and economic research within our own business that we have gradually developed a fairly definite and tangible picture of requirements as to statistical personnel. It is not my intention to direct your consideration to the statistical problems and requirements which are peculiar to the telephone organization. In the general field of business, statistical activities, whether their scale be large or small, must depend almost wholly for their characteristics upon the type of business enterprise with which they are associated. The statistical problems of a corner grocery do not resemble either in scope or in kind the statistical problems of a great merchandising corporation; the statistical problems of the merchandising corporation do not necessarily resemble those of the manufacturing organization or the great public utility. Nevertheless, there are certain essential elements which must characterize statistical analysis in whatever field of business it appears. These elements are, first, the collection and tabulation of facts, second, the analysis and comparison of these facts and deduction from them of sound conclusions of practical value, and, third, the presentation of these conclusions in such a manner as to result in their conversion into actual policies of management and operation.

In the small business concern it will frequently happen that business analysis which is statistical in spirit and in method is conducted by the executives themselves; and in the large concern much of this same analysis may be classified as engineering. The man called "Statistician" may merely be the man who collects and tabulates facts, any application to be made of these facts devolving entirely upon men higher in the organization. But it does not follow that this accidental limitation of title represents in any sense a limitation of the true function of statistical analysis. By whatever name it is known within a business organization, statistical analysis has not served its full function until the principles and conclusions developed by it have been converted into policies of administration.

More than fifty years ago, Oliver Wendell Holmes in "The Poet at the Breakfast Table" attempted a classification which I find particularly illuminating in any consideration of personnel requirements. Oliver Wendell Holmes may never have heard of a statistician, and he certainly had no picture of the present day development of business analysis. But he made the following observation: "One-story intellects, two-story intellects, three-story intellects with skylights. All fact-collectors, who have no aim beyond their facts, are one-story men. Two-story men compare, reason, generalize, using the labors of the fact-collectors as well as their own. Three-story men idealize, imagine,

predict; their best illumination comes from above, through the skylight."

The parallel is clear. Dr. Holmes' one-story men—fact collectors who have no aim beyond their facts—represent unfortunately an ancient and still somewhat popular view of the statistician. Fact collectors are necessary, and statistical method has much to teach them as to procedure in the collection of their facts. But the statistical program which stops at this point in a present day business organization is not worthy of the name. We must have the one-story men; we must also have two-story men who can "compare, reason, and generalize," who, treating their facts honestly and dispassionately, are able to reason objectively to the inevitable consequences of these facts. Even with the addition of two-story men, however, the statistical function is not yet completely realized. We must have the three-story men who, building on the facts of the one-story men and the conclusions of the two-story men, are able to idealize, to project their conclusions into the future, to shape fundamental policies of business administration in such fashion that, so far as possible, the largest elements of hazard may be eliminated and a successful future for the business enterprise be assured.

You may say that I am describing the business executive. I am. The young men most needed in the field of business statistics today are the men who can exhibit, in however embryonic a form, those innate qualities of mind and personality that give promise of eventual attainment to executive rank.

This broad interpretation of the statistical function has been frequently presented and is without doubt well understood by the more important business organizations. It is not well understood, however, by the general public and I have frequently had occasion to doubt whether it is fully appreciated by the majority of teachers of statistics in the colleges and universities. I say this because of what seems to me an over-emphasis upon the mechanics of fact collection and tabulation. In my own contact with college graduates who are seeking employment, I am constantly impressed with the number who believe that their qualifications for the field of statistical research are adequate because they happen to have completed a half-year or a year of the usual courses in business statistics. It goes without saying that the collection and tabulation of facts involve the observance of certain statistical principles and the application of a certain well recognized statistical technique. But to assume that a man is ready to undertake statistical analysis for the sole reason that he has done satisfactory work in an elementary course in business statistics is about as illogical as to

assume that a medical student is ready to undertake major surgery because he happens to have been successful in a first course in anatomy.

I hope that I may not be misunderstood. It is not my intention to decry or even to minimize the importance of elementary courses in business statistics. I merely wish to point out that, in my judgment and in the judgment of many employers with whom I have discussed this subject, courses in business statistics must be placed well down near the end of a list of essential qualifications for successful business statisticians.

To prepare a list of essential qualifications that would receive universal approbation by employers in the business field is probably impossible. Even if such a list could be prepared, I am skeptical whether the statistician exists today who could completely satisfy its requirements. There are certain outstanding items, however, in which I believe we can find general concurrence.

You will agree, for example, that a statistician must possess the so-called "old time virtues" of industry, loyalty, and integrity. He must know how to serve faithfully the interest of his superiors and the organization of which he is a part. He must know how to coöperate with the men of his own rank. He must know how to win the confidence and inspire the loyalty of the men under him. These attributes, to be sure, are generally important in every type of business activity. But the fact that they are not peculiar to the statistical field does not mean that they should be disregarded. No matter what his technical qualifications, the success of the business statistician will be limited if he is unfortunate in personality. In addition to these personal attributes, the business statistician should have a general foundation of economic training. He must also know how to write lucid and forceful English. And, when opportunity arises, he must be able to defend his conclusions in such a manner as to instill confidence and carry conviction.

A man may satisfy all of these specifications, however, and still not attain the level of Dr. Holmes' "three-story man." There are qualities of mind which I have left to the last for the purpose of emphasis. They are the qualities of mind for which employers of statisticians in the business field are today most eagerly searching. They have been stated and re-stated in a variety of forms. I think they can be summarized briefly as curiosity, imagination, and power of application. By curiosity I do not mean, of course, the casual and idle curiosity of the non-productive mind. I mean scientific curiosity, that same unquenchable scientific curiosity which characterizes the productive research worker in the field of physical phenomena. By imagination,

I mean an imagination so disciplined and controlled that it is directed into practical channels and that even before data are collected the result of an analysis may be visualized in order to determine whether it will be of such practical value as to justify the necessary cost of time and money. By power of application, I mean that quality which Ex-President Eliot of Harvard has defined as a "serviceable mind." "A serviceable mind," to quote his own definition, "is the mind capable of concentration, of an intense application to the task in hand. That power of application, that power of concentrating all your forces, all memory and reasoning on the task of the moment, is the principal thing you ought to get."

It may be said that these latter qualifications—curiosity, imagination, and power of application—are not susceptible of precise definition, that no course of training however comprehensive can insure their development in a student, that no tests however detailed can determine the precise degree to which an applicant satisfies these requirements. This is true. On the other hand, given a student with the proper native propensities, there is an educational discipline which will aid materially in his development. I refer in particular to the continued and intensive study of mathematics and the physical sciences.

It must not be thought that training in mathematics and the physical sciences is desirable only because of the informational content of these subjects. There is, to be sure, a large and rapidly growing field for the mathematical statistician in business. Speaking again for the telephone business, this field is already a very great one. The mathematical statistician must not only meet the requirements which I have already outlined for the general business statistician but, in addition, must be equipped with an intensive comprehension and mastery of certain branches of advanced mathematical research.

Entirely apart from those considerations, however, I think we must recognize the function of mathematics and the physical sciences as an educational discipline. All science has for its end and aim "the classification of facts, the recognition of their sequence and relative significance," and the formulation of sound and unbiased judgment. Mathematics and the physical sciences are characterized by the very type of analytical and sequential thinking for which the business statistician has need. I do not mean that mathematical and other scientific training is a necessary prerequisite to success in the field of business statistics. Any investigation of the academic training received by the outstanding business statisticians of today would quickly indicate the absurdity of such a notion. I am thinking primarily in terms of the young man who is planning a college course which he hopes

will give him the best possible training for such a career. To him and to his academic counselors I say, there is no realm of education which presents a greater opportunity for training in rigorous thinking and disciplined imagination.

Our job, whether as teachers of statisticians or as employers of statisticians, is to get for ourselves a proper appreciation of the full scope of statistical analysis in the field of business, to the end that we may secure and train the type of men required. "One-story intellects, two-story intellects, three-story intellects with skylights." The one-story men are necessary in a statistical organization, but no college student whose equipment as to mind and personality is such as to prohibit his development beyond that stage should ever be encouraged, either by employer or by teacher, in the hope that he may find a happy career in the field of business statistics. Courses in statistics may be of material value to the one-story man; they alone cannot make him become a successful business statistician. We must attract to this field more of the potential two- and three-story men. We must encourage their training, both in the university and in the business organization itself, along lines best calculated to develop those qualities of mind which we believe to be essential.

III

CONTENT AND PURPOSE OF TRAINING IN ELEMENTARY AND IN ADVANCED STATISTICS

By Willford I. King

All too commonly the actions of those responsible for the university curriculum in statistics have seemed to conform to the description: "We don't know where we're going, but we're on our way;" at least in many instances the program for statistical instruction shows no evidence of having been laid out according to a logical plan. Apparently the tendency has been to overlook entirely the necessity of framing each course to accomplish some specific purpose, and the fact that the needs of various classes of students for statistical training differ widely from each other never even seems to have been sensed. To speak in the language of economics, the supply of courses has not been planned with any regard to the nature of the demand.

The ordinary school feels that it is doing well when, to cover the needs of all classes of students, it provides a single course in elementary statistics. If, however, we grant the premise that every course should be designed to attain some definite objective, logic compels us to

admit that any single course, no matter how well taught, must be inadequate unless the majority of students interested are in need of practically the same kind of statistical training. What are the facts in this connection?

Students who ought to take statistics may be divided into two chief classes:

1. Those who will be called upon merely to make use of published data, but who, nevertheless, will need to know both where to look for reliable figures and how to judge their merits. Most students of science fall in this group.
2. Those who intend to adopt statistical work as a profession, and hence, in addition to being able to analyze critically the work of others, will need to become thoroughly familiar with methods of handling raw data. This group is small as compared with the former one.

Of course, those of us who specialize in statistics can scarcely help feeling that it is essential for every intelligent person to be thoroughly trained in statistical technique in order that he may be safeguarded from being led astray by statistical fallacies. That such training is desirable cannot be denied, but the fact should be frankly recognized that this goal is unattainable. A great variety of interesting subjects beckon to the student, and most of the members of the faculty, outside of those who happen to teach statistics, are doing their best to see that he has little time left for this new branch of mathematics. The usual result is the installation of a rather easy compromise course—one which skims lightly over the entire field by means of perhaps two lectures or recitations a week for one semester, with very little laboratory work.

COURSE IN THE INTERPRETATION OF STATISTICS

It is my contention that there is room in nearly every college curriculum for an easy introduction to statistics, but that such a course should be frankly described as giving training merely in the Interpretation of Statistics, and not disguised, as is so frequently done at present, under the title Elementary Statistical Method.

This interpretative course should familiarize the student with some of the leading official sources of statistical information such as the Census, the Statistical Abstract, and the reports of the Comptroller of the Currency, the Department of Agriculture, the Bureau of Labor Statistics, and the Interstate Commerce Commission. Opportunity should also be given to learn something about the work of private

agencies dealing in financial statistics and about the investigations being carried on by research bureaus. The student should be taught how to interpret the usual types of charts, and stress should be laid on the commoner fallacies against which he needs to guard himself.

To aid him in this work of interpretation he should learn the chief characteristics of the median, the mode, and the arithmetic average, both simple and weighted. He should be instructed briefly in the meaning of relatives and index numbers and should be warned against attaching too much significance to coefficients of correlation. The greatest emphasis of all, perhaps, should be laid upon the seriousness of the errors arising from failure to define with accuracy either the problem or the statistical units used, or growing out of relative comparisons obtained by aid of the wrong numerators or the wrong denominators.

At first thought it may seem that the outline just stated covers a large part of the field of elementary statistics—and in a way it does—but there is a vast difference between giving the student a general understanding of the meaning of these concepts and making him master of the technique required to use them constructively. For example, it is much easier to explain the meaning of a straight line trend, fitted by the method of least squares, than it is to teach the student to fit the line himself. However, it is by no means certain that practice in fitting the line will help in the least toward an understanding of the nature of the concept involved.

In an interpretative course of this nature laboratory work of the painstaking type may be reduced to a minimum, though a certain amount is highly desirable. To say that formal laboratory work is non-essential is, however, radically different from saying that lecturing on the subject is adequate. My experience, both as a teacher and as an employer of statistical help supposedly trained by the lecture method, indicates that, as far as the students are concerned, the lecturer on statistics is wasting his breath; in fact, lecturing is no more effective as a means of teaching statistical method than it is for giving instruction in any other branch of mathematics. The truth is that the methods settled upon for instilling the fundamentals of algebra or geometry are equally applicable to the teaching of elementary statistical method.

No class in statistics ought to contain more than twenty or thirty students. Plenty of blackboard space should be available, and, during a large proportion of the time, the class should be kept busy solving statistical problems of the simplest types. The rest of the recitation hour should be devoted to quizzing on the text or illustrating statistical principles by concrete examples.

ELEMENTARY COURSE IN STATISTICAL TECHNIQUE

For the second class of students mentioned—namely, those who expect to engage in statistical work as a profession or to conduct research on their own account—the training required must be far more intensive than is necessary for the group just described. A mere general knowledge of the principles involved is of little use when one is called upon to deal with an unsorted mass of figures. The beginning course adapted to the needs of the prospective statistician should familiarize him thoroughly not only with the underlying principles but also with the technique involved in going through with fundamental statistical processes.

There is no particular reason why the nature of the class work given to this group should differ materially from that used with the classes in statistical interpretation. The prospective statistician will likewise need drill on the principles laid down in the text and plenty of blackboard practice. But he will require something in addition if he is really to become competent to handle figures, namely, a large amount of laboratory work. He should find out how to start with a mass of raw data and bring order out of chaos. He should learn to prepare not only working tables and charts but also those suitable for publication. He should overcome the difficulties involved in computing averages, measures of dispersion, seasonals, trends, and coefficients of various kinds. He should become familiar with a number of ways of presenting facts graphically and should learn which method is best fitted to bring out the comparison that he wishes to emphasize.

As you all know, the ends just mentioned cannot be obtained without much practice. My experience indicates that, to cover the bare essentials, even in a hasty way, requires that the student spend in the laboratory at least eight hours each week for a semester. I firmly believe that this is the minimum necessary to give anything like an adequate foundation for further training.

Undoubtedly, to many students this laborious method of learning the principles of statistics seems very unattractive. Isn't there some way to make the course easier and more popular? Greek history tells us that the young Alexander asked this same question about geometry some two thousand years ago, and every generation since has vainly repeated it. The answer still remains the same and applies as well to statistics as to geometry!

Instruction in statistical interpretation may well be adapted to the general run of students. Everyone can use to advantage the type of training given in such a course. Nothing seems more foolish, however,

than to attempt to popularize the study of statistical technique. The records of the Employment Clearing House of this Association indicate a surplus rather than a shortage of partially trained statisticians. The school that turns out from its courses in methods students possessed of but a smattering of the subject, not only injures its own reputation but also does a gross injustice both to the well trained student seeking a position and likewise to the employer.

Many instructors believe that the way to teach students statistical technique is to assign them real investigations possessing considerable complexity and involving numerous statistical processes. They point out rightly that this method of instruction has the marked advantage that it tends to hold the student's interest far better than does the assignment of a series of brief artificial problems.

All this is true, but this plan of teaching has a disadvantage which, in my opinion, is so great as to condemn the method—it is extremely wasteful of the student's time. No matter how carefully the investigation is planned, it is almost certain to be of such a nature as to force the student to devote most of his time to a rather limited number of phases of statistics. I believe there is no university course in the country which allows sufficient time for laboratory work to cover by this method in one semester nearly all of the subjects discussed in the most elementary of our textbooks. As you all know too well, statistical work is abominably slow and tedious at best. To cover the ground, it is therefore necessary to make problems brief, and this can best be accomplished by using for each topic data carefully selected to bring out the essentials with the least possible amount of effort.

ADVANCED COURSE IN STATISTICAL TECHNIQUE

So much for the training in elementary statistical method. The courses in advanced method should begin where the one just described leaves off. Emphasis should continue to be laid upon the fundamental principles. Once the principles are thoroughly mastered, the applications will largely take care of themselves. There is but little point to training the student in agricultural statistics because he expects to work in agriculture, or in financial statistics because he hopes to get into a bank. The principles are the same in both fields, and if he understands the fundamentals he will have no difficulty with the applications. The best place to learn the technique pertinent to a particular business is on the job and not at school. Only the large university should bother at all with courses in applied statistical method.

COURSES IN INFORMATIONAL STATISTICS

Of course, there is no limit to the number of helpful lecture courses that may be devoted to informational statistics. These are but indirectly related to statistical method. I have found, for example, that students become much interested in such a course in economic statistics; but while lectures of this type help the student to become more proficient in economics, history, or geography, they improve but incidentally the knowledge of these same students concerning statistical method. Such courses belong in the fields to which the information pertains. Statistical method is, of course, merely a branch of mathematics.

My conclusion, then, is that the improvement most generally needed in the realm of instruction in statistical method is the differentiation between the popular interpretative treatment and the serious technical course. The next most important need is a great increase in the intensity of training in the latter course.

IV

MATHEMATICAL BACKGROUND FOR THE STUDY OF STATISTICS

By H. L. Rietz and A. R. Crathorne

We are to open a discussion on the mathematical background necessary for technical training in statistics. Our Program Committee for this meeting very appropriately divided the consideration of this background into two parts:

(1) Elementary and intermediate training,
(2) Advanced statistics and specific research.

While there seems to be no sharp line of separation between these parts, the division will facilitate the discussion because it has been the practice of some of us to give two courses in statistics corresponding more or less closely to the two kinds of training indicated in the classification. Two courses are offered because it has seemed to be good educational policy and fair economy to recognize two classes of students with respect to their qualifications to enter upon the study of statistics. The one group may consist very naturally of the rather mature undergraduate students, say juniors and seniors, and graduate students from departments such as biology, economics, psychology, education, and sociology. These students should acquire an understanding of the elements of statistical methods as part of the scientific method they are attempting to use in all these lines of study, but they are often lacking almost entirely in a knowledge of college mathematics. The other

group may consist of students who have had college mathematics through at least a thorough first course in the calculus, and have the same degree of maturity as the other group. The two classes of students thus recognized would probably include no freshmen or sophomores. While it seems that some courses in statistics are open to sophomores, and even to freshmen, and while there may be circumstances making this desirable, it is here submitted for your consideration that ordinarily students would do better through the sophomore year to spend their time on mathematical preparation for statistics to be taken in the junior or senior year than to take statistics as early as the sophomore year. With these introductory remarks on the division of the subject, we proceed to consider more directly the mathematical background for the "elementary and intermediate" training. We shall not distinguish between elementary training and intermediate training but simply consider them as differing in degree.

Considerable light is thrown on the necessary background for a first course in statistics by the numerous introductory textbooks which have appeared recently. It seems that each of these books is designed to serve as a basis for teaching certain statistical methods with a minimal mathematical background. In fact, it seems fairly obvious that the authors of some of these very useful textbooks would have made their books more mathematical if the students and readers for whom the books are intended, were prepared to study more mathematical books. This plan of avoiding mathematics as much as possible has the advantage of reaching more readers than could be reached with a more mathematical treatment. In certain of these books, it is explicitly stated that the book aims to meet the requirements of students who have not had college mathematics. In one excellent book, it is assumed that the readers know elementary algebra. That is, "the reader must have more than a reminiscence of the ground covered in elementary algebra" and must know logarithms. In some of the books, a short treatment of logarithms is given. These books are no doubt well adapted to give an understanding of certain very useful statistical methods with a minimum of mathematical background.

It seems, therefore, that from the point of view of understanding enough of statistical methods to be worth while, our question as to the necessary mathematical background may be answered by saying that a working knowledge of elementary algebra would serve the purpose. But it is not unlikely that such an answer will raise an argument if we are considering the man who is looking forward to making statistics his profession. It may, however, facilitate the present discussion if we take the rather arbitrary position that the ele-

mentary and intermediate training are designed for the understanding of certain processes of statistical analysis rather than for the training of the statistical specialist, and turn to the advanced statistics to complete our discussion. Before leaving the elementary course of training, it should probably be said that much more can be accomplished in a given time in teaching even an elementary course to students with good mathematical training than to students without such training. Experience has impressed this fact on some of us over and over again when we have happened to have in an elementary course a few students who have college mathematics as a background. On this account it is the very natural reaction of the mathematically minded teacher to feel that we should solve the problem of mathematical background by requiring some training in college mathematics as a prerequisite for any course in statistics. Such a view may be theoretically sound but perhaps not very practical because it would greatly limit the number of persons to whom certain elementary statistical methods would be available. On the other hand, the teaching of statistical operations without some college mathematics as a background is not without its drawbacks. It is interpreted by some as technical training. Such training is likely to lead to an undesirable kind of formality and industry in calculating averages, standard deviations, correlation coefficients, and other statistical constants. The mathematician rarely errs on the side of formality in dealing with an applied problem; it is much more likely to be those who have learned a formal mathematical process without understanding the mathematics or the limitations on the interpretation of the formulas. For instance, some years ago one of the writers gave some lectures on correlation before students and instructors in agriculture. Presently it was found that one of the instructors was so given to correlating things that the dean of the college was asked in a semi-jocular manner to stop this man "correlating." A college mathematics instructor who is interested in the training of future statisticians is supposed to give much more emphasis to the mathematical side than is really necessary. He is supposed to believe and teach that mathematics takes the place of common sense and knowledge of the subject matter of the statistical problem. There probably are such instructors, but it has been the experience of both of us that blind faith in mathematics is more likely to come from one who knows little mathematics. One of the functions of the teacher of the mathematical side of statistics is to teach the limitations of mathematics as well as its uses. This can be done to some extent with nothing but a course in elementary algebra behind it. The trained statistician who has aspirations to cover a broad field

needs more than this, and the necessary mathematical background for technical training in statistics should include much more mathematics than elementary algebra.

Turning now to advanced statistics under which the mathematical background necessary for technical training is to be considered, we need first a criterion as to the meaning of advanced statistics. It must be recognized that much valuable work has been done in statistical analysis of important material by common sense methods of classification and analysis that depend mainly on extensive knowledge of the field from which the data are drawn. The statistical methods in such valuable work may be simple and elementary, although much knowledge, experience, and broad scholarship are necessary in the analysis and interpretation of the data. The work may thus be of an advanced nature, but not with respect to the statistical methods employed. For convenience in the present discussion we shall assume that satisfactory training in advanced statistics involves a high degree of facility in reading substantially all the literature of descriptive and mathematical statistics, and the ability to apply the methods involved to concrete data.

With this aim before us it is fairly obvious that at least a thorough first course in calculus is essential to enter upon the study of advanced statistics, and that it would facilitate progress if much more were included in the mathematical background. Indeed, we do not get far in a course designed to carry out such aims before we need a much more extensive mathematical background. Thus, we soon need least squares in fitting a regression line or curve unless we include this topic in the course. We soon need determinants. We need finite differences in the background in the use of interpolation and approximate summation, unless the subject is included as part of the statistics course. We need definite integrals not treated in a first course in calculus. For example, we soon come upon the normal probability integral, such definite integrals as the gamma functions, and the incomplete gamma functions in considering the Pearson frequency curves. Here we also come upon another definite integral

$$\int_0^{\pi} \sin^r \phi \, e^{v\phi} \, d\phi$$

in treating Pearson's type IV. While the values of this integral are tabulated, the underlying theory for the valuation would take us into the function theory of a complex variable. Next, in the Gram-Charlier representation we have to deal with infinite series. The con-

vergence of the series requires consideration. In certain demonstrations in the literature these series are integrated term by term. The validity of this process is dependent on the uniform convergence of the series. We thus find ourselves dealing with questions of fundamental mathematical reasoning, because of the infinite processes involved. In the recent papers of Wicksell we have need for a knowledge of partial differential equations as part of the background. In thus surveying the mathematics involved in statistics our background begins to look like a fairly extensive mathematical program and we wonder where to draw the line.

Assuming for the present that we are looking forward to the training of a professional class of technical statisticians similar to actuaries or civil engineers, it may be possible that a reasonable background will be suggested by examining the mathematical background of actuaries and engineers, and for an extreme case the scientific background of recently trained physicians. The actuarial societies of this country require in their examinations for admission a mathematical background which may be described as college mathematics through a first course in the differential and integral calculus, a course in higher algebra beyond the calculus, a course in finite differences, and the elements of statistics. The knowledge of these subjects is tested by four searching examination papers written in four sittings amounting in all to twelve hours. While the literature of actuarial science involves some papers for which this ground is inadequate, the training seems well adapted to reading the bulk of the literature. It is a fact that a few men are successful as actuaries without belonging to an actuarial society, but the number of such men is relatively small.

Turning next to the civil or electrical engineer, there are certain engineers who know very little mathematics and who nevertheless render valuable service as engineers. They ordinarily depend much on the application of certain formulas in handbooks for engineers. It is, however, not likely that any one would defend the thesis that it is economy and good educational policy for a young man to omit calculus and analytic mechanics from his training if he plans to be an engineer. It is not true that every engineer will use the mathematics often, but this knowledge is such a necessary background that it is the part of reasonable educational policy to have every engineering student take such training.

In advising young men as to their preparation to deal with new problems, there is always great danger that they will omit the background. Medical education of today compared with that of thirty years ago furnishes striking illustration of the introduction of a back-

ground. The medical student of today does not observe patients the first day, month, or year, of his course. He observes test tubes and microscopes instead. The medical student not only spends two years as a minimum in a college of liberal arts, studying physics, chemistry, animal biology, but after he enters the college of medicine he studies more organic chemistry, more bio-chemistry, anatomy, pharmacology and so on.

In conclusion, may we not well grant on the one hand that courses in statistics with elementary algebra as a mathematical background are desirable for the purpose of making the meaning of statistics and the scientific method involved available to a larger group of persons, but urge on the other hand that it is good educational policy to make the minimum mathematical background for technical training in statistics at least equal to that of the actuary, for the subject of statistics is mathematically somewhat more complex in its advanced aspects than actuarial theory. To some this may be a very disproportionate demand on the part of mathematics. Many statisticians will differ from us. They can give many instances of men who have done brilliant work in statistics without such a mathematical equipment. They can also give many instances of men who have had such equipment who have failed miserably. We consider these the extreme cases. A man who has taken a well planned course in college, covering courses in mathematical statistics as well as general courses in statistics, may never use much, if any, advanced mathematics, just as many engineers never do, but he will not find a large part of the field of statistics so walled in that he is unable to enter it. We are not saying that the highest type of statistician is he who can handle mathematics best but we do say that bravery and poise in the presence of such things as integral and summation signs, infinite series, beta and gamma functions are not drawbacks.

V

STATISTICS IN INSURANCE SERVICE

By Edwin W. Kopf

Interest in statistical training for actuaries, accountants, and general statisticians in insurance service has been re-kindled by the papers and discussion recently presented to the Institute of Actuaries of Great Britain.[1] British actuaries have been more and more inclined

[1] Menzler, F. A. A., "Future of the Actuarial Profession with Some Remarks on Extending its Scope," *The Review*, London, October 30, 1925. Also, *Economic World*, New York, November 21, 1925, p. 742; *The Review*, London, November 10, 1925; *Journal of the Institute of Actuaries*, March, 1926, p. 88.

toward statistical work ever since 1906 when Mr. W. Palin Elderton introduced actuaries the world over to the methods of Karl Pearson and his contemporaries. A number of trained actuaries are now serving as vital and social statisticians in the civil services of several of the Commonwealths in the British Empire. This tendency of actuaries in Great Britain to enter statistical work has become stronger in recent years, since the growth in numbers of the actuarial profession in that country during the past thirty years has not been accompanied by a corresponding increase in opportunities for the employment of full-fledged actuaries in the higher ranks of the companies' services. Two steps seem to face British actuaries: the first, a change of policy regarding admission of new, young members to the profession; or second, to maintain the open door by extending the work of the actuary in fields outside of insurance and to dissipate certain mistaken notions as to the narrow exclusiveness of the profession. In other words, it is proposed to broaden the training of actuaries so that opportunities for employment in statistical service may be embraced by the actuary in fields of public administration, industrial, commercial, economic, and political research. These developments ought to interest teachers of statistics.

In the following remarks I shall abide by the published syllabi of the established actuarial societies which prescribe courses of study for actuaries, accountants, and statisticians in the insurance business. The requirements may seem rigid as drawn by practitioners in the business; the mathematical instruction in two universities preparing students for actuarial and statistical work is, however, far more comprehensive than the practicing actuary would deem necessary. In the words of one experienced teacher of insurance science, "a professor (of actuarial science) usually is guided by other notions or different notions as to what should be the mathematical groundwork of the (actuarial) student than the examiners who ordinarily set the questions in professional societies."

Within the insurance business there are numerous opportunities for applying to good effect the full range of elementary, intermediate, and advanced aspects of the calculus of observations, which contains the theory of probabilities, "statistical method," or whatever other name we may apply to methods of inductive analysis of quantitative data. In addition to analytical method, wide use is also found in the insurance business for the precepts of crude statistical description—the collection, classification, tabulation, graphic illustration, and the assembly of crude numerical data. High competence in the management of crude descriptive statistical processes is imperative because the

major portion of the budget for actuarial, accounting, and statistical work is devoted to the conduct of routine record work.

Actuaries, accountants, and general statisticians in the insurance business use part or all of these descriptive and analytical procedures. In addition, the general statistician acts as custodian for generalized data—the body of compiled and published external data on the political, economic, social, and hygienic status of mankind.

We shall have space to consider only the educational problem for actuaries and accountants, and for statisticians in the general administrative service of the life insurance companies. Something could be said, also, on the training of general officers not in these three technical services. Statistical training is only one element, however, in the formal discipline of these technicians. What, therefore, are the general requirements for the student entering actuarial, accounting, or the general statistical service of the companies, and to what extent does training in statistics enter into this program? This question can be answered in the light of the syllabi constructed by the educational committees of the several actuarial societies in the United States and abroad. These syllabi outline the subjects in which generalized competence is expected of the candidate for professional standing in the business. Exceptional competence in one or a few of the subjects is expected within the special field in which either the actuary, the accountant, or the general statistician is employed.

It is assumed that the future supply of candidates for these positions in life insurance service will come from the colleges and universities. An outline of the subjects in which general competence is required will be of interest not only to teachers of statistics and insurance, but to students who contemplate an insurance career. While the preliminary requirements are rather exacting, the program of study is no more difficult than that required in a number of other professions (engineering, architecture, etc.). It may be well to examine existing training programs for students of actuarial science, accounting, and statistics in the insurance business.

I. *What the colleges and universities can do for the student-actuary*

(a) UNDERGRADUATE STUDIES

A survey of the syllabi of the actuarial societies suggests that perhaps the most useful program of undergraduate study for the student-actuary would have the following general complexion: A general "science" course emphasizing mathematics (advanced algebra and through the differential calculus, the introductory mathematics of

investment and finance, and, specifically, the financial mathematics of insurance); physics, chemistry, and general biology, with plenty of laboratory work in order to form concepts of law and causation; logic; a reading knowledge of two modern languages; introductory economics; introduction to the law, and commercial law; elements of accounting; an introductory course in money and banking; the principles of insurance, with description of the more important lines of coverage; two modern languages; and an introduction to descriptive and analytical statistics.

This covers the academic or preparatory syllabus of the Casualty Actuarial Society recently adopted by the Council of that Society. It is by no means so comprehensive or so difficult as an undergraduate course for engineers, and may be taken as a statement of minimal requirements. It may be augmented by subjects transferred from the schedule of graduate studies outlined below.

(b) GRADUATE STUDIES

The following list of subjects is not meant to apply to the graduate work of a single student. Some of the subjects may be given to undergraduate students. It is assumed, also, that a judicious selection will be made according to the temperament and taste of the student and in accordance with the time available for the pursuit of graduate studies. It seems desirable to list in a program of graduate work the following subjects for choice: Mathematics (differential and integral equations; the theory of equations; determinants; finite differences; the calculus of observations, including the theory of probabilities; the principles of graduation, interpolation, and curve fitting, and practical laboratory work in that part of the calculus of observations which is sometimes known as advanced statistical method); the history, methods and substance of vital statistics; insurance law; substantive economics; money and banking; insurance investment history and practice; insurance history; insurance accounting; problems in insurance supervision, regulation, and taxation; practical rate-making, valuation, and surplus distribution procedure in life insurance. This covers technical study. Teachers of graduate students in insurance may wish to broaden such instruction culturally in the manner outlined by Professor Pearl.[1]

Following academic preparation of this general type, the candidate for actuarial service would be prepared, after suitable coaching during his introduction to the business under an experienced actuary, to pass the examinations of the actuarial societies and thus to secure

[1] "The Reading of Graduate Students," *Scientific Monthly*, July, 1925.

professional standing. The syllabus may seem to be overloaded with mathematics. It should be remembered, however, that the pursuit of discipline by actuaries in specialized insurance mathematics is not an end in itself; it constitutes only one of the many branches of insurance science which the actuary must master. It is evident that the educational programs of the actuarial societies provide for a liberal discipline in insurance science and that there is no narrow interpretation of the scope of an actuary's work.

II. *What the offices are doing for the student-actuary*

Some of the larger companies provide instruction for the students preparing for the examinations of the actuarial societies and often give direction to the work of the student-actuary, so as to afford wide contact with the problems of the business.

III. *What the colleges and universities can do for the student insurance-accountant*

(a) UNDERGRADUATE STUDIES

Primarily, the training should follow the accepted programs which university schools of business have outlined for students of accounting. In addition to a sound course in introductory or elementary statistical method this program would cover: Accounting (principles of accounting; accounting problems; auditing; investment accounting and insurance accounting); introductory economics; introduction to the law, and commercial law; an introductory course in money and banking; investments; principles and description of the leading lines of insurance coverage; the mathematics of accounting and finance; an introductory course in management principles; two modern languages. This program for the accounting student is no heavier than that prescribed in many university schools of business.

Preparation in these subjects will enable the candidate to pass the examinations of one of the actuarial societies admitting accountants to membership under the syllabus as recently revised.[1]

(b) GRADUATE STUDIES

This would cover courses in advanced auditing; problems in the compilation and construction of insurance annual statements and exhibits; an examination of existing insurance accounting plans, and research which would lead to a codification of insurance accounting principles and methods; studies in budgetary control; insurance taxa-

[1] Syllabus of the Casualty Actuarial Society, with Recommendations for Study. Prepared by the Educational Committee and approved by the Council of the Society, 1925.

tion; insurance cost-accounting; insurance investments; the statistical dissection of the expense schedule; the principles and methods of report construction and the graphical analysis of insurance accounting data.

IV. *What the offices are doing for the insurance student-accountant*

No effort is being put forth by the offices at the present time. A preliminary step has been taken by the Casualty Actuarial Society to provide a text on casualty insurance accounting.[1] It is hoped that other actuarial societies will help later on in the developing of an accounting text for other branches of the business. In these texts, some consideration should be given to the elements of statistics, emphasizing the utility of mechanical aids to accounting, and the use of graphical and other methods of analysis of value to the financial executives of the company. A text on life insurance accounting has recently been issued by Mr. Earl C. Wightman with the coöperation of Professor James W. Glover.[2]

V. *What the colleges and universities can do for the student of insurance statistics in administration service*

(a) UNDERGRADUATE STUDIES

For the student who hopes eventually to meet the demands of the administrative service of the business, the same course of undergraduate study should be prescribed as for the student-actuary. This recommendation follows the syllabi of actuarial societies which admit statisticians to membership. Some departure may be made, however, from the schedule of graduate studies outlined for actuaries, in order to meet the practical demands upon statisticians in the general administration service.

(b) GRADUATE STUDIES

The same requirements in graduate studies hold for the group of student-statisticians as for student-actuaries, with the exception that extended and highly technical treatment of rate-making, valuation, and surplus distribution problems may be omitted. In view of the demands upon general statisticians in insurance service, it may be well to substitute for the subjects omitted, several of the following: Anthropometry; public hygiene; social insurance history and practice; political, legal, or industrial history; the economic theory of risk and risk bearing; management theory and practice; or historical studies of insurance

[1] Being prepared by Mr. R. S. Hull, Travelers Insurance Company, Hartford, Connecticut, with the assistance of a special advisory committee, Mr. George D. Moore, Chairman.

[2] *Life Insurance Accounting*, Edwards Brothers, Ann Arbor, Michigan, 1925.

operations. Some part of this graduate schedule could be transferred to undergraduate work for selected students.

The statistician may be expected to administer a general information service, to conduct historical and critical studies of insurance operations, to handle problems in the general domain of insurance economics and insurance public health practice, coöperate with the other technicians in the business in the conduct of service-studies, and in other ways to act as counselor in matters which call for the "aptitude for vicariousness." The general statistician should be qualified to apply in his studies of insurance operations the many rigorous rules of evidence which test the validity of conclusions based upon numerical data and which lead to an offer of choices of action to the executives.

VI. *What the offices are doing to provide training for the student-statistician in insurance service*

After entering the business, students are encouraged to study the several branches of insurance science with the exception of the highly technical actuarial branches. Several years of actuarial discipline for student-statisticans have been found to be of service, however.

Study classes are sometimes formed in insurance economics, insurance history, management, and administration, and in statistical mathematics. There is no reason why study groups should not be encouraged to treat successively insurance accounting; the origins in accounting and investment records of scheduled items in the annual statement; practical problems in insurance supervision, regulation, and taxation, chiefly those which have an historical basis; insurance law, including the discussion of recent court decisions and legislation respecting insurance operations; etc. Any effort which promotes understanding of the complex functioning of an insurance organization and of the relations of insurance institutions to society is praiseworthy and adds to the efficiency of such centralized statistical service as may, in the judgment of the executive, seem to be necessary.

AN HYPOTHESIS CONCERNING THE DURATION OF BUSINESS CYCLES

By Frederick C. Mills, *National Bureau of Economic Research*

A recent publication of the National Bureau of Economic Research[1] contains the following figures relating to the duration of business cycles in various countries:

TABLE I

AVERAGE DURATION OF BUSINESS CYCLES IN SEVENTEEN COUNTRIES

(1) Country	(2) Period	(3) Total number of cycles	(4) Average duration of cycles (in years)
United States	1796–1923	32	4.9
Japan	1890–1920	7	4.3
Russia	1891–1925	7	4.9
South Africa	1890–1920	6	5.0
Argentina	1890–1920	6	5.0
Brazil	1889–1924	7	5.0
Canada	1888–1924	7	5.1
Germany	1848–1925	15	5.1
India	1889–1920	6	5.2
France	1838–1920	15	5.5
Austria	1866–1922	10	5.6
Sweden	1892–1920	5	5.6
Australia	1890–1924	6	5.7
England	1793–1920	22	5.8
Netherlands	1891–1920	5	5.8
Italy	1888–1920	5	6.4
China	1888–1920	5	6.4

From his study of the data upon which the above averages are based Dr. Mitchell is led to this conclusion: "The materials appear to be homogeneous, with (this) important exception—the distribution of American business cycles in respect to length differs from the distribution of cycles in other countries."

This conclusion raises one specific question: Why does the distribution of American cycles in respect to length differ from the distribution of cycles in other countries? The figures in the preceding table present a wider problem: Why does the average duration of cycles differ from country to country? Neither of these questions may be answered in terms of final causes. It is the purpose of the present paper to suggest the direction in which an explanation may lie.

The hypothesis tested in this paper is that the duration of business cycles in a given country is a function of the stage of industrial development which that country has attained. More specifically: When the modern type of economic organization is in the initial stages of

[1] *Business Annals* by Willard L. Thorp. The figures quoted are from Wesley C. Mitchell's introductory chapter.

development, the average duration of cycles is relatively long. During the stage of rapid growth, when modern types of business enterprise and modern forms of industrial organization are being applied extensively, business cycles are of relatively short average duration. With the decline in the rate of economic change and the attainment of comparative stability, business cycles increase again in length.

The *rationale* of this hypothesis takes the following form:

1. The business cycle is a characteristic of the typically modern form of business and industrial organization. Business cycles proper did not occur under simpler forms of economic organization.
2. The first stage in the transition to the distinctively modern economy is an era of slow alteration. Certain industries and regions may be changing rapidly under the impetus of the new methods, but the national economy as a whole is still dominated by the practices and attitudes of the earlier type of economic life. Business cycles first appear during this stage, but the cumulation of those forces which tend to terminate a period of prosperity, as of those which tend to bring about revival after depression, requires a comparatively long period of time.
3. The second stage is one of rapid growth for the industries of the country as a whole. The extensive application of new methods of management and production offers rare opportunities to organizers, and there is a scramble to exploit these opportunities. Under these conditions the maladjustments which lead to crises develop more quickly, and there is a corresponding speeding up in the recovery from depression.
4. The third stage is one of approximate stability, marked by a decreasing rate of economic change. The new system has displaced the old, to the extent which the resources of the country and the character of its people permit.[1] With this decline in the rate of economic change there are fewer of those conjunctural opportunities for profits which abound in the stage of rapid transition. Stresses within the business system accumulate less rapidly and the forces of revival operate more slowly than during the period of exuberant growth.[2]

[1] This condition of relative stability may come when a very large percentage of the population is engaged in industrial pursuits, as in England, or it may come while a country is still predominantly agricultural. By the last quarter of the nineteenth century France appeared to have reached such a state of stability, but without being as fully industrialized as England.

[2] The general reasoning advanced above might also support a theory concerning changes in the intensity of cyclical fluctuations, a theory which would be generally similar to that relating to the duration of cycles. The evidence to support or disprove such an hypothesis is lacking at present, however.

If the above hypothesis be valid, the data of the *Annals* should permit statistical verification. There should, in the first place, be evidence that the duration of the cycle in a given country changes with the economic development of that country. Secondly, a cross section of a number of countries in varying stages of economic development should reveal differences in cycle duration of the type suggested by this hypothesis. Finally, if all recorded cycles be divided into three groups, the classification being based upon the stage of economic development in which the individual cycles have occurred, we should expect to find significant differences between the groups in respect to cycle duration.

Two difficulties present themselves. In the first place, we have no objective criterion for distinguishing the stages in a country's industrial development, or for classifying countries according to their present state of development.[1] An attempt at such a classification is made below, but it is quite experimental and tentative.

The second difficulty is a familiar one in statistical studies—the presence of a considerable degree of variation in the observations. In a single country and at a given period successive cycles vary widely in duration. Accordingly, any marked secular change in the factors affecting the duration of cycles would be apparent only in the average length, not in the length of any individual cycle.

A first test of the hypothesis is provided by figures showing the duration of cycles in individual countries,[2] at different stages of their development. It is unfortunate that we have no single record covering all the phases of industrial development, but for certain countries the data cover intervals of time sufficiently long to permit subdivision, and comparison of the averages for different periods. Dividing the recorded number of cycles roughly into thirds, we have the following averages for four countries:

[1] It would be a work of considerable economic value to construct an index by means of which the rate of change in the conditions of economic life at different periods might be measured, for given countries. This might be termed an *index of industrialization*, though the process in question is broader than is suggested by that expression. The percentage of the population employed in manufacturing establishments is one possible, though probably inadequate, measure. The prevalence of *exchange*, as reflected in the volume of trade or the velocity of circulation of money, would perhaps serve as a better indication of the degree of development of that form of economic organization within which business cycles are generated. Since a comparison of countries would involve a comparison of rates of change, not of absolute values, identical original measures would not be needed for all countries.

Such an index would be very useful in the study of certain aspects of economic change. R. A. Lehfeldt ("The Normal Law of Progress," *Journal of the Royal Statistical Society*, 1916) has suggested that social and economic change follow some such course as that outlined above, but the theory has not been adequately tested. This hypothesis concerning variations in the rate of economic change accords in a general way with Raymond Pearl's theory of population growth (*The Biology of Population Growth*, New York, 1925).

[2] *Business Annals*, pp. 42–49.

TABLE II

AVERAGE DURATION OF BUSINESS CYCLES, BY PERIODS, IN ENGLAND, FRANCE, GERMANY, AND THE UNITED STATES*

(1) Country	(2)	(3)	(4)
	Average duration of cycles, in years		
	First period	Second period	Third period
England	4.57	5.86	6.78
France	5.60	4.60	6.00
Germany	5.80	5.25	4.20
United States	4.45	4.12	3.27

* The years covered by these periods, for the various countries, are as follows. The number of cycles occurring in each period is given in parentheses, after the dates.

	First period	*Second period*	*Third period*
England	1793–1825 (7)	1825–1866 (7)	1866–1920 (8)
France	1838–1867 (5)	1867–1890 (5)	1890–1920 (5)
Germany	1848–1878 (5)	1878–1904 (5)	1904–1925 (5)
United States	1796–1845 (11)	1845–1890 (11)	1890–1923 (10)

Though these periods have been quite arbitrarily chosen, definite tendencies toward changing average durations are apparent in the figures. In England the tendency has been toward an increasing length, in the United States and Germany there has been an apparent decline in the average length, while the French figures register first a decline and then an increase.[1] These movements, considered in connection with the industrial history of these countries, accord very well with the hypothesis advanced above.

But such casual comparison of averages yields no clear-cut conclusions. We must answer this question: Are the differences between the averages for a given country at varying periods great enough to indicate real differences in the forces affecting the duration of cycles at different times? If the presence of such real differences could be established, it would serve as conclusive proof that there are secular changes in the underlying factors upon which the duration of cycles depends. In seeking evidence of such changes we may make somewhat different classifications of cycles in each country, selecting periods which appear to be homogeneous, as judged by the cycle lengths recorded. The following averages have been computed for England, France, and the United States:[2]

[1] Dr. Mitchell has called attention to the above tendencies in England and Germany, and to the tendency to increase toward the end of the French period. With respect to the United States his conclusions were somewhat different. Judging from averages for the two halves of the period covered, he concludes that the average length of cycles has remained constant in this country. The particular classification here employed indicates a secular decline in cycle lengths. More is said on this point below.

[2] Germany has been omitted because the changes in cycle duration in that country, though present, have not been as pronounced as for the three countries included in the table. As will later appear, most of the years covered by the German records seem to fall within a single homogeneous economic era.

TABLE III

AVERAGE DURATION OF BUSINESS CYCLES, BY PERIODS, IN ENGLAND, FRANCE, AND THE UNITED STATES *

(1) Country	First period		Second period		Third period		Fourth period	
	(2) Number of cycles	(3) Average duration (years)	(4) Number of cycles	(5) Average duration (years)	(6) Number of cycles	(7) Average duration (years)	(8) Number of cycles	(9) Average duration (years)
England	9	4.22	13	6.85	..	....	..	....
France	6	3.67	7	6.32	..	....	..	....
United States	5	5.20	11	3.50	5	5.50	11	3.20

* The years covered by these periods, for the various countries, are:

	First period	*Second period*	*Third period*	*Fourth period*
England	1793–1831	1831–1920		
France	1854–1876	1876–1920		
United States	1796–1822	1822–1860	1860–1888	1888–1923

There is definite evidence, in the English averages given above, of real differences between the factors affecting the duration of business cycles in the two periods named. The chance that the observed difference between 4.22 years and 6.85 years is due to sampling fluctuations alone, and not to the operation of a true cause, is much less than 1 out of 100.

For France the proof of a real secular change in cycle duration is not conclusive. In about one case out of 20 a difference as great as that observed, or greater, might arise as a result of sampling fluctuations.

The classification of American figures shows a curious cyclical movement in the duration of cycles. A period of relatively long cycles during our early history was followed, between 1822 and 1860, by a succession of short cycles. After 1860 came five long cycles, and these were succeeded by a series of cycles having the shortest average length recorded in any period of equal length for any country whose history is covered by the *Annals*.

The differences between the averages for the first and second American periods and for the second and third American periods are of about the same order of significance as the difference found in France. There is a strong indication that there were changes in the factors at work, but since chance would bring about differences of this magnitude in, roughly, one trial out of 20, we cannot definitely say that chance was not responsible. The change was more marked after 1888, however. A difference as great as that found between the averages for the third and fourth periods (5.50 and 3.20) would arise because of chance fluctuations less than one time out of 50.[1]

[1] While these results must be accepted with some reservations, because of the nature of the original measurements and the smallness of the samples, the observed differences cannot be ignored. The fact that these differences accord with a prior hypothesis gives them added significance.

A cross-section view of the 17 countries included in the *Annals* furnishes a second test of the hypothesis, for these countries represent widely different stages of industrial development. In Table IV are averages measuring the mean duration of the business cycle in these countries within recent years. This table differs from Table I in that the periods covered by the data for different countries are approximately equal.

TABLE IV

AVERAGE DURATION OF BUSINESS CYCLES IN SEVENTEEN COUNTRIES

(The period covered begins with the recession nearest to 1890 and ends with the last recorded recession)

(1) Country	(2) Average duration of cycles (in years)	(1) Country	(2) Average duration of cycles (in years)
China	6.4	Italy [1] (since 1907)	4.2
Australia	5.7	United States	3.3
India	5.2	Germany	5.0
Argentina	5.1	Austria	5.1
Canada	5.1	Sweden	5.6
Brazil	5.0	Netherlands	5.9
South Africa	5.0	England	5.9
Russia	4.9	France	6.0
Japan	4.3		

[1] The one exception with respect to the period covered was made in the case of Italy. During the entire period from 1888 to 1920 there were five cycles in Italy with an average duration of 6.4 years. It is believed that a more representative average can be secured by taking account only of the period from 1907 to date. There is reason to think that 1907 represents the approximate date of a fairly definite economic transition. This turning date is suggested by W. C. Mitchell, in connection with another problem. ("The countries whose business cycles diverge most from the international pattern are Italy before say 1907, Russia, South Africa, Brazil and China—all countries rather backward in economic organization and predominantly agricultural." *Business Annals*, p. 93.) The Italian average as given is, of course, based upon too small a number of cycles to be accepted as thoroughly representative.

The arrangement of countries in this list is based upon an estimate of their probable place in the process of industrialization. Since the averages are objective facts, the only room for the exercise of judgment in making up this list is in determining whether a given country should fall in the first half or in the second half of the table.

Accidental factors must, of course, affect the averages over such a short period, but in spite of this the figures accord reasonably well with the hypothesis suggested. At the upper end of the list, with relatively long cycles, we find those countries which have come most

The application of the usual statistical tests for determining the significance of a difference between averages has, I think, full justification. As Dr. Mitchell points out, the data bear evidence that the duration of each cycle is the resultant of a multiplicity of causes which are at least semi-independent. Only if this were the case would we find, as Mitchell does, a distribution of cycles according, within the limits of sampling, with the logarithmic normal law. It is certain that there is some degree of interdependence between cycle lengths in different countries, but this relationship would tend to make the observed differences more significant than is here assumed.

The samples available are small, when the groups are limited to observations in single countries, but the theory of errors for small samples has been worked out with sufficient precision to justify its application here. (For a discussion of this theory see "The Probable Error of a Mean," by "Student," *Biometrika*, 1908; "On the Distribution of the Standard Deviations of Small Samples," by Karl Pearson, *Biometrika*, 1915. A general treatment of the theory, with useful tables, appears in R. A. Fisher's *Statistical Methods for Research Workers*, London, 1925.)

recently under the sway of business motives, and which, in their economic life, are still largely dominated by the conditions and processes of a simpler economy. China, Australia, and India stand at this end of the scale. Passing down the list, the average cycle length decreases until the turning point on the scale is reached at the United States, with an average duration of 3.3 years. Japan and Italy are closest to this country. Below the United States are six western European countries, with average cycle lengths varying from 5 to 6 years. The Netherlands, England, and France, all countries in which the modern form of economic organization has long been dominant, stand closely bunched at the foot of the list.

In all the above comparisons the samples employed have been small. Stronger proof may be drawn from larger groups. Such larger samples may be obtained by the division of all the cycles recorded in the *Annals* into three classes—those which have occurred in the initial stages of industrialization, those occurring in the stage of rapid transition, and those occurring after the rapid transition has been completed. While the individual cycles falling in each of these three classes may be expected to differ materially in duration, the average for the second class should be significantly smaller than the averages for the two other classes.

The difficulty in the application of this test has been suggested. We have no thoroughly objective criterion by which we may distinguish the different stages in a country's economic development. In the absence of a suitable criterion a somewhat arbitrary division into periods has been made, a division based in part upon the evidence of changing conditions which the data themselves yield. To some extent, then, the argument is circular. The following classification has other justification, however. The dates refer, of course, to periods of change which can seldom be precisely fixed in time. The given years represent approximations only.

In making such a classification the division of each country's history into periods must be based upon that country's own economic development and not upon comparison with other countries. To say that the United States reached a turning point in its economic development about 1822, while Germany reached such a turning point about 1866, and Italy about 1907, is not to say that economic conditions in these countries at these dates were similar. The dates are comparable only in the sense that they mark the beginning of eras of fairly rapid industrial change in these several countries. Similarly, it is not to be assumed that conditions in countries at the same stage of development are identical. England and the Netherlands, in

A. Countries in the early stages of industrialization:

United States	to 1822	(Annals begin 1796)*
Germany	to 1866	" " 1848
Italy	to 1907	" " 1888
Canada	to 1913	" " 1888
Australia	to 1913	" " 1890
South Africa	to 1913	" " 1890
China	to date	" " 1888
India	to date	" " 1889
Russia	to date	" " 1891
Argentina	to date	" " 1890
Brazil	to date	" " 1889

B. Countries in the stage of rapid economic transition:

England	to 1831	(Annals begin 1793)
United States	1822 to date	
France	to 1876	" " 1838
Germany	1866 to date	
Austria	to 1873	" " 1866
Italy	1907 to date	
Canada	1913 to date	
Australia	1913 to date	
South Africa	1913 to date	
Japan	to date	" " 1890

C. Countries in which the transition is going forward at a decreasing rate:

England	1831 to date	
France	1876 to date	
Austria	1873 to date	
Netherlands	to date	(Annals begin 1891)
Sweden	to date	" " 1892

*The dates given as marking the beginning of the *Annals* are the dates of the first recorded recessions. In cases where the beginning of a stage is not defined in the above table, that stage is assumed to date from a period prior to the beginning of the *Annals*.

recent years, have been classified together as being in the third stage of development. This means, merely, that in both countries innovations and changes in business methods and industrial processes have ceased to be the dominating feature of their economic life.

Since the general classification is based upon the rate of economic change and the stability of conditions existing during a given period, no absolute criterion can be employed for international comparison. For example, Sweden has a larger proportion of its population engaged in agricultural operations than has Australia, while France has a larger proportion than Germany. Yet there is good reason for classing Sweden and France among those countries which have attained approximate economic stability, while Australia and Germany may be classed as still in the stage of transition. (Economic readjustments due to the war may affect the rates of economic change in these and other countries, but more time must elapse before this can be determined.)

The above classification has been tested to determine whether the various subordinate groups of cycles combined in constructing each of the three distributions are themselves homogeneous. For each

distribution the constituent groups of cycles drawn from different countries are statistically similar, so far as one may judge from the averages. For example, the group of English cycles occurring between 1793 and 1831, the American cycles occurring between 1822 and 1923, the Japanese cycles occurring between 1890 and 1920, and all the other groups of observations in distribution B, constitute a single homogeneous group. This is in rather remarkable contrast to the fact that the English cycles occurring between 1793 and 1831 and those occurring between 1831 and 1920 do not, in combination, constitute a homogeneous group.

The length of the period of transition in the United States calls for some comment. This stage is assumed to have continued for over 100 years in this country, and the closing date has not yet been reached. The corresponding English and German figures are smaller. The beginning of the era of transition in England goes back beyond the beginning of the *Annals*, but it might be set at about 1770. This would give the period of fairly rapid change in England a length of about 60 years. To date, the industrial transition in Germany has run about the same length of time. Reasons for the prolongation of the period of transition in the United States may be found in the geographical extent of this country, and in its industrial diversity. The constant pushing out of the industrial frontier and the opening of new territories to economic exploitation would tend to make the conditions peculiar to a period of economic transition endure for a longer time. A smaller and economically more homogeneous country might be expected to pass through the stage of transition a great deal more quickly. There is in this the suggestion of a solution of the problem raised by Dr. Mitchell and noted at the beginning of this paper: Why does the distribution of American cycles in respect to length differ from the distribution of cycles in other countries? An exceptionally prolonged period of industrial transition and a speeding up of the cycle during this transition would have just the result noted by Mitchell—an average length distinctly shorter than that found in other countries.[1]

A classification of the cycles occurring in these different industrial stages, in different countries, yields the frequency distributions in Table V.

[1] It is possible that a truer account of the industrial development of the United States would be obtained if it were assumed that this country has passed through two cycles of economic growth, marked by two distinct stages of industrial acceleration. The first of these stages of acceleration extended from about 1820 to 1860, and was followed by 30 years of less rapid change, with greater relative emphasis on agriculture. The second and most pronounced period of industrial acceleration began about 1890 and has continued to the present date.

TABLE V

CLASSIFICATIONS OF BUSINESS CYCLES OCCURRING DURING THREE STAGES OF INDUSTRIAL DEVELOPMENT

(1) Duration of cycles (in years)	(2) Number of cycles during Early stages A	(3) Number of cycles during Middle stages B	(4) Number of cycles during Late stages C
1	..	3	..
2	4	10	2
3	6	22	3
4	6	15	5
5	7	12	3
6	7	8	7
7	10	2	5
8	4	2	6
9	3	2	2
10	2	1	3
11	1	..	2
12	1	..	..
Totals	51	77	38

From these distributions the following measures are derived:

	Mean length of cycle (in years)	Standard deviation (in years)
Distribution A (Including cycles during early stages of industrialization)	5.86	2.41
Distribution B (Including cycles during period of economic transition)	4.09	1.88
Distribution C (Including cycles during stage of economic stability)	6.39	2.42

A difference as great as that found between the averages of distribution A and of distribution B would arise as a result of sampling fluctuations about one time out of 50,000 trials. A difference as great as that found between the averages of distribution B and of distribution C would arise as a result of chance about one time out of 1,000,000 trials. Which is to say that these differences could not possibly be due to sampling fluctuations. If the above classification of countries according to stages of industrial development has validity, and if the data of the *Annals* regarding cycle lengths may be accepted as accurate, there is proof here of a definite secular change in the factors determining the duration of business cycles.[1]

[1] This conclusion holds even though considerable alterations be made in the schedules defining the stages of economic development in the different countries. For example, we may assume that the era of rapid transition extended to 1857 in England, and that this era began in Australia and Canada about 1900. The differences between the averages of the three distributions are slightly reduced, if these changes be made, but the odds against chance furnishing an explanation of the differences are, for practical purposes, just as strong.

SUMMARY

1. There is presented an hypothesis that the duration of business cycles is a function of the stage of industrial development. This function is such, it is contended, that during the early stages of industrialization in a given country the average duration of cycles is relatively long, that there is a distinct shortening of the average duration during the period of sharp industrial and economic transition, and that during the era of relative stability which follows this transition the average duration of cycles again increases.
2. Supporting evidence of three types has been adduced.
   a. There have been found significant differences between averages measuring the duration of cycles at different periods in the history of individual countries. These differences confirm the fact of a secular change in cycle duration.
   b. Figures showing the average duration of cycles in 17 countries, for the period since 1890, support the general hypothesis. The cycle has been shortest in the United States, the country in which the pace of industrial development has probably been most rapid during these years. The average duration has been longest for China, at one extreme, and for the Netherlands, England, and France at the other.
   c. A classification of all recorded cycles, according to the stage of industrial development in which the individual cycles are assumed to have occurred, reveals differences between the group averages much too great to be attributed to chance. Granting the validity of the original classification, these measures give definite evidence of secular and regional differences in the forces determining the duration of business cycles. These differences accord with the hypothesis outlined above.
3. It is suggested that the relatively short average duration of American cycles is due to an exceptional prolongation of the period of industrial transition in this country. This prolongation may be due to the geographical extent of the country, and to its industrial diversity.
4. Although there is definite evidence of a secular change in cycle duration in individual countries, complete verification of the hypothesis here advanced will require the application of some objective criterion in defining the stages of economic development in given countries.

ARRESTS AND DEATHS SINCE 1920 ATTRIBUTABLE TO ALCOHOL[1]

By Walter F. Willcox

The question what effects are the Eighteenth Amendment and the Volstead Act having upon the habits of the American people is one upon which most diverse opinions are entertained and confidently expressed. The evidence in support of these opinions is far from conclusive. Many years must pass before a final verdict is reached. Meantime what evidence appears deserves careful analysis.

Back of and obscured by the clash of opinion at the recent committee hearings in Washington a certain amount of weighty evidence was presented. It was presented by propagandist organizations but by organizations taking opposite views of the facts or of the inferences from them. In its light I wish to examine two questions, first, what effects the Eighteenth Amendment and the Volstead Act have had in New York State upon the number of arrests for drunkenness and upon the number of deaths from alcoholism and cirrhosis of the liver, and, secondly, what effects they have had upon the number of arrests and of deaths from those causes in the other parts of the country for which the information exists. There are two reasons why New York State has been chosen for the main study. It is the most populous state in the Union and the evidence about it is more abundant and comes down nearer to the present day than that for any other state. In later paragraphs the inquiry will be extended to the rest of the country in order to meet the obvious objection that New York does not afford a fair test because neither state sentiment nor state legislation in New York supports the new Federal policy as well as they do elsewhere.

1. What effects have the Eighteenth Amendment and the Volstead Act had in New York upon the number of arrests for drunkenness?

These arrests are not compiled and published by any state office.

[1] As this note goes to press I have received from the Moderation League a later tabulation containing the figures of arrests for drunkenness in 1925, but I see nothing in it requiring a change in my argument or conclusions. I have also just received from Professor Fisher a copy of his "Prohibition at its Worst," in which he criticises at length the figures of the Moderation League, accepts those of the Anti-Saloon League as the "standard," and endorses its research secretary as "a careful statistician." From the testimony and documents presented by the two organizations I see little reason to prefer either set of figures to the other, or to regard either secretary as careful or judicial in his interpretations. Both are statisticians working for propagandist organizations. Both seem to me honest and untrained partisans. But nothing in Professor Fisher's argument materially affects mine, for, as I have shown elsewhere, he is wrong in saying that the figures given by the Moderation League for Indianapolis "appear to have no official support," and has failed to establish his assertion that other figures from the same source are unsafe to use because they are "inaccurate and unverifiable."

To learn about them one must apply to the city police departments. In 1920 New York contained 22 cities each with 25,000 inhabitants or more. A circular letter was sent last year to the municipal police departments of these cities asking for the number of arrests for drunkenness in each year since 1914. Replies were received from 20 cities, all but Amsterdam and Syracuse, covering each year from 1917 to 1924, and the answers were presented to the Senate Committee. There is no reason to suppose that statements for Amsterdam and Syracuse, if they could be included, would change the general result.

The following figures are of the annual arrests for drunkenness in the larger cities of New York since 1917, and the ratio of those numbers to the low point in 1920.

Year	Annual arrests for drunkenness in 20 large New York cities	Ratio to figure for 1920
1917	49,034	230
1918	34,016	160
1919	23,835	112
1920	21,303	100
1921	24,743	116
1922	30,707	145
1923	40,896	192
1924	39,903	187

There was a steady drop in the number of arrests after 1917 to a minimum in 1920, when the Eighteenth Amendment and the Volstead Act became effective. After 1920 the increase in the number of arrests was almost as rapid as the previous drop.

It is not quite fair, however, to add the figures for the different cities. The ratios of arrests to population differ so widely as to show that the enforcement of law or, more probably, the record and classification of arrests differ widely in different places. So it is better to treat each city separately and trace the fall and rise in the number of arrests by computing the ratio of the arrests in each city in 1914, 1917, and 1924 to the low point in the same city. Ten cities had the minimum in 1920; 5 in 1919; 4 in 1921, and 1 in 1918. When the minimum was not in 1920 its date is indicated after the figure in the third column. In 7 of these 20 cities the number of arrests in 1924 was greater than in 1914 and in 8 it was greater than in 1917. In 10 of the 20 cities the increase of arrests from the low point to 1924 was more and in 10 it was less than 118 per cent, which may therefore be regarded as the average increase in large New York cities. This is greater than the average of 87 per cent obtained by the other method because the latter is pulled down by the slight increase of arrests for drunkenness in the populous cities of New York and Buffalo.

City	1914	1917	Minimum	1924
Albany	465	497	100	863
Auburn	842	377	100 ('19)	423
Binghamton	143	269	100 ('19)	175
Buffalo	187	194	100	152
Elmira	372	488	100	183
Jamestown	266	258	100 ('19)	362
Kingston	?	561	100	848
Mt. Vernon	?	144	100 ('21)	175
Newburgh	186	194	100 ('19)	250
New Rochelle	198	269	100 ('21)	215
New York	336	235	100 ('19)	190
Niagara Falls	129	274	100	116
Poughkeepsie	375	377	100	319
Rochester	407	255	100	211
Rome	458	783	100 ('21)	477
Schenectady	207	251	100	258
Troy	?	250	100	211
Utica	308	321	100	219
Watertown	260	481	100 ('21)	217
Yonkers	293	161	100 ('18)	338
Median	293	269		218

In these cities the total number of arrests for drunkenness in 1925 was less by nearly 3,000 than in 1924. This was due to a marked decrease in New York City and Buffalo; in the other cities combined there was a slight increase. If each city, large or small, be given the same weight, a method which probably throws most light on the urban trend in the whole state, the average or median number of arrests in 1925 increased over 1924 a little faster than the population.

There are also 27 New York cities having between 5,000 and 25,000 inhabitants for which the arrests in each year 1914–24 are reported. The totals and ratios for these small cities are given below:

Year	Arrests for drunkenness in 27 small New York cities	Ratio to minimum
1914	7,158	327
1915	6,973	319
1916	8,345	381
1917	7,899	361
1918	5,087	232
1919	2,529	116
1920	2,185	100
1921	2,896	132
1922	3,647	167
1923	5,876	269
1924	6,185	283

The drop in arrests between 1917 and 1920 in these small cities was much more rapid than in the large ones. In the small cities the number of arrests went down to about one-fourth and then in the following four years nearly trebled. These figures suggest, but are far from proving, that in the small cities the supply of intoxicants was more completely cut off by national prohibition but that since it went into effect the clandestine and illegal trade has developed in them more rapidly.

Is this tendency confined to New York cities or does it appear elsewhere?

Massachusetts is the only state for which complete figures are available. Here too the low point was reached in 1920. Calling that 100, the ratios for the other years were:

Year	Ratio	Year	Ratio
1914	291	1920	100
1915	285	1921	160
1916	314	1922	204
1917	348	1923	227
1918	250	1924	231
1919	213		

Arrests for drunkenness considerably more than doubled between 1920 and 1924. It is noteworthy, however, that in the New York and Massachusetts figures the increase in the last two years was at a much slower rate than in the first two, suggesting that in these states, at least, arrests for drunkenness may be approaching a constant number.

Figures for arrests for drunkenness in 128 cities in 38 states outside of New York State and Massachusetts were also presented to the Senate Committee. The number of arrests ran from 90,776 in 1920 to 231,154 in 1916. The ratios were as follows:

Year	Ratio	Year	Ratio
1914	231	1920	100
1915	233	1921	141
1916	255	1922	197
1917	237	1923	224
1918	185	1924	232
1919	146		

All the evidence points to a high point in arrests for drunkenness reached in 1916 or 1917, a low point in 1920, and an increase after 1920, at first very rapid and then slackening as if stabilization at approximately the 1914 number is at hand.

In judging the significance of these figures it must be remembered that the cities have grown and that consequently the ratios of arrests to population have increased more slowly than the arrests. But in view of the uncertainty of all estimates of city population, governmental or private, to compute rates in this case seems an unnecessary refinement.

To the main inferences from these figures that public intoxication is almost but not quite as common in the United States as it was in 1914, and that it is slowly increasing, several objections have been made.

1. Being issued by a propagandist organization and for purposes of propaganda, the figures are untrustworthy.

2. They speak only for cities and not even for all cities.

3. They neglect the frequency with which the same person is arrested for drunkenness and so furnish little light upon the number of different persons so arrested.

4. The figures for the most recent years indicate that the increase has been checked or turned into a decrease.

5. Both inferences depend upon the wrong assumption that the proportion of arrests to drunken persons in public places has not materially increased since prohibition went into force.

On these objections a few comments may be made:

1. The figures have been gathered by two organizations—The World League Against Alcoholism and The Moderation League. Their views about policy are sharply opposed and yet neither has accused the other of falsifying or suppressing the facts. The figures, therefore, may probably be trusted.

2. The evidence for the entire state of Massachusetts does not differ noticeably from that for the cities alone in other states.

3. A count in one small city where I have access to the records showed that the annual number of persons arrested for drunkenness was about three-fourths the number of arrests for that cause.

4. This objection is a weighty one. The figures indicate that the increase has slackened since 1923.

5. Arrests for drunkenness today are said to be a far better measure of the amount of public intoxication than the arrests of ten years ago or more when the police were more lenient. There is probably some truth in the opinion, but how much I cannot say. The number of police has increased much faster than the population: the kind of liquor drunk has changed and a drunken person is probably more dangerous than formerly, both to the public peace and to his own family or friends; the multiplication of automobiles and their use in the transportation of liquor have greatly increased the number of drunken drivers. On the other hand, clandestine drinking and drunkenness may have increased with the clandestine sale. It may be that the amount of public intoxication has decreased since 1914 much more rapidly than the arrests for drunkenness. Even if this be so, it does not affect the gravest point in the situation, which is the rapid increase of arrests after the low point was reached. That increase indicates that since 1920 a clandestine and illegal trade in intoxicants has developed and is reaching a larger proportion of the population.

Another line of evidence, that of deaths from alcoholic poisoning, avoids several of these objections. The causes of death are stated as

a routine professional duty by the physicians of the country and are classified, tabulated, and published by the Federal government. The figures apply to country districts as well as cities and cover a population between three and four times as large, more than four-fifths of the population of the United States. Complications due to repeated arrests of the same person and to increased vigilance and efficiency of the police do not arise.

The figures of deaths from alcoholic poisoning, however, are open to other objections. A physician often hesitates to certify that a death was due to alcoholism, even when he is confident that it was the real cause. A high English authority has estimated that the true number of deaths from alcoholism is about six times the reported number. However that may be, there is little doubt that the published figures greatly understate the facts. Still it is probable that this error is approximately the same from year to year and so that the trend of the reported figures is a trustworthy index to the trend of the much larger actual figures.

Because of the understatement in the deaths ascribed to alcoholism, the deaths from cirrhosis of the liver are often said to be the best measure of deaths from alcoholic poisoning; in this case, however, the figures are too large. Probably more than half of the deaths from cirrhosis of the liver are due to alcoholism, but many are due to syphilis. On the whole, the best measure of the mortality from alcoholic poisoning is perhaps to be found by adding the deaths from alcoholism to those from cirrhosis of the liver. Even these figures are probably too small because the deaths from alcoholism which are not so certified probably outnumber the deaths from cirrhosis of the liver which are not alcoholic in origin. The results for New York State are as follows:

	Deaths from alcoholism	Cirrhosis	Both causes
1900–04 (annual average)	488	1,328	1,816
1905–09 " "	611	1,591	2,202
1910–14 " "	599	1,628	2,227
1915–19 " "	501	1,342	1,843
1920	123	872	995
1921	164	956	1,120
1922	309	997	1,306
1923	469	912	1,381
1924	625	1,028	1,653
1925*	733	1,104	1,837
1926**	837	1,107	1,944

* The figures for 1925 and 1926 are from the tabulation of the State Department of Health and in the case of cirrhosis of the liver have been increased by one-third because the Federal figures in the rest of the column regularly exceed the state figures in that proportion owing to some unexplained difference in classification.

** On basis of figures for the first eight months of 1926 and assuming no increase over 1925 in the rest of the year.

The preceding figures indicate that there has been a steady increase in deaths from each cause in New York State since 1920, that the increase in deaths from alcoholism has been much more rapid than that from cirrhosis of the liver, and that the deaths from the latter cause are less common and those from alcoholism more common than before 1920. The figures also suggest what other evidence supports, that drinking habits in the community have been rapidly changing; that steady drinking to excess, such as often develops cirrhosis of the liver, is less common and occasional debauches using concentrated and low grade alcoholic stimulants are more common than formerly. They indicate that both causes of death are still increasing but that the rate of increase is slackening.

The evidence from the rest of the country will indicate whether New York State is exceptional. It seems best to include all states which have been since 1920 in the registration area for deaths, established by the Federal Bureau of the Census. These states include more than four-fifths of the country's population. Returns from them do not in all cases go back of 1920 and the figures for 1925 have not yet been published. The results are as follows:

DEATHS IN THE REGISTRATION STATES OF 1920 CERTIFIED AS DUE TO

Year	Alcoholism	Cirrhosis of the liver	Both causes
1920	873	6,102	6,975
1921	1,573	6,453	8,026
1922	2,406	6,698	9,104
1923	3,044	6,667	9,711
1924	3,036	6,931	9,967
	Ratios		
	100	100	100
	180	106	115
	276	110	131
	349	109	139
	348	114	143

In the country at large the deaths from alcoholism have increased in four years between three- and four-fold; the deaths ascribed to cirrhosis of the liver have a little more than kept pace with the increase of population; from both causes combined the increase has been about two-fifths. The figures for alcoholism suggest that the increase in deaths from that cause ended in 1923, but the New York figures for 1925 and 1926 make the inference doubtful.

One main conclusion seems well established: there has been throughout the country an increase in the arrests and the deaths from alco-

holism since 1920, an increase which at first was very rapid and later was more gradual but has not yet ended.

Are we not learning in our own way what the experience of other countries might have taught us had we been in a teachable mood, "that great laxity and the multiplication of facilities for obtaining intoxicants tend to increase drinking and disorder and that extreme severity produces the same or worse effects by driving the trade into illicit channels which escape control and thus really increase facilities while apparently diminishing them"?

NOTES

A NEW COMPOSITE INDEX OF WAGES IN THE UNITED STATES

By Carl Snyder

In this Journal for June, 1924, account was given of a new index of the general average of all kinds of payments in exchange in this country, or, as it was termed, of the General Price Level. This had been worked out in view of the fact that, especially in the war period and after, our familiar indexes of commodity prices at wholesale had, in the violence of their ups and downs, obviously ceased to be representative or adequate measures of average purchasing power or the value in exchange of money. This was due to the fact, first, that even the best of these indexes represent simply current price quotations and not actual sales prices, and as very interesting studies by Walter R. Ingalls, Joseph H. Barber, and others have shown, in times of rapid price movement quotations may rise while the actual volume of sales is falling; second, that in such periods wholesale prices always fluctuate much more widely than retail prices which must, of course, also include rents, wages, and handlers' profits; third, because even the best of our commodity price indexes are, in the view of the writer, too heavily weighted (one-half and more) with food and farm products to be representative of any general average of payments or prices.

Furthermore, it was evident from our new measures of the velocity of bank deposits and from the volume of trade that the equation of exchange could not be satisfied, in this period, even approximately by an index of commodity prices as representative of P.[1] In any broad composite it was clearly necessary to include for a considerable weight an index of wages, since through these particular years at least, the average of wages had varied so widely from the average of wholesale prices; and since wages have now become such a large part of the actual volume of payments in exchange for goods, services, and property.

The index of wages used was a composite of the best materials then available, viz., New York State factory wages, clerical wages, teachers' salaries, and an index of unskilled workers' wages prepared by the

[1] Cf. Carl Snyder, "New Measures in the Equation of Exchange," *American Economic Review*, December, 1924, p. 699.

Research Department at the Federal Reserve Bank of New York. The aim was, of course, to secure as representative a cross section of wages and salaries as could be obtained.

Since this index was made up, valuable new material has become available. The Interstate Commerce Commission has begun the publication of a monthly index of railroad wages; the Agricultural Department has collected the data for a quarterly index of farm wages: and the National Association of Builders Exchanges has supplied the material for a monthly index of builders' wages, made up by the Federal Reserve Bank. As this material covered the earnings of a very large number of workers not hitherto available, it seemed desirable to recast our composite to include this added information. This has recently been done and the seven series have been weighted in proportion to the number of workers reported as engaged in the several lines of industry, as nearly as could be ascertained. Upon this basis the weighting for the new index is as follows:

Agricultural wages	5
Railroad "	10
Teachers' "	5
Factory "	40
Building "	15
Clerical "	15
Unskilled labor "	10

The accompanying charts give a comparison between our old and new composites, and of these with the earnings of New York State factory workers and the components of the new index. The main differences between the old index and the new, both being based on 1913 taken as 100, are that the new begins on a slightly lower level in 1919, due largely to the inclusion of building wages; does not rise quite so high at the peak of 1920; does not fall quite so low in 1922 nor rise quite so high in the middle of 1923; but has been rising more continuously since.

But the differences are really slight and suggest that even if it were possible to obtain an average of all wages paid in the United States, the picture would still remain practically the same. In these seven groups the wage data cover several million workers, highly representative of the wage earners of the country.

It will be seen that, interestingly enough, taken from the low point at the close of the War, the highest percentage of rise has been in teachers' salaries, next in building wages, and third in clerical wages. Taken on a 1919 base, agricultural wages are now much lower, railroad

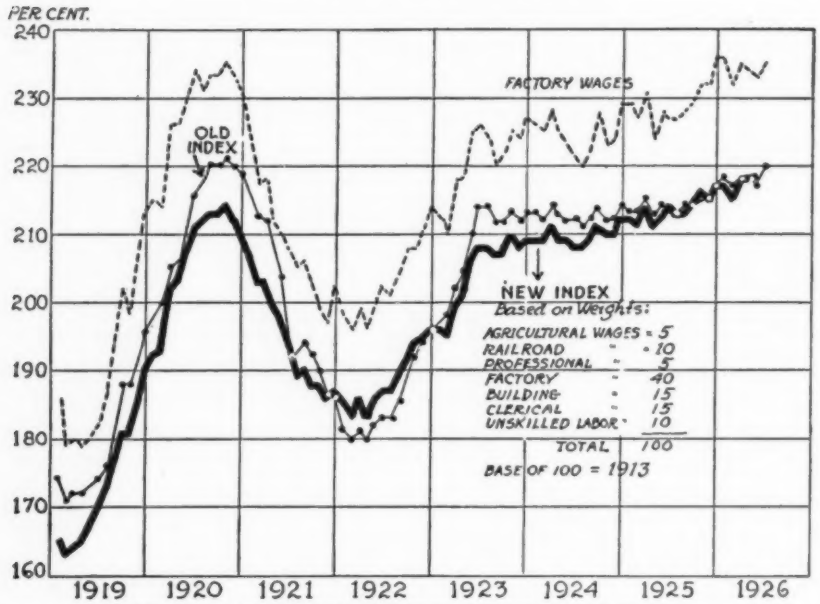


CHART 1. A NEW COMPOSITE INDEX OF WAGES IN U. S.

and unskilled labor wages just a little higher, and average factory wages about 20 per cent higher.

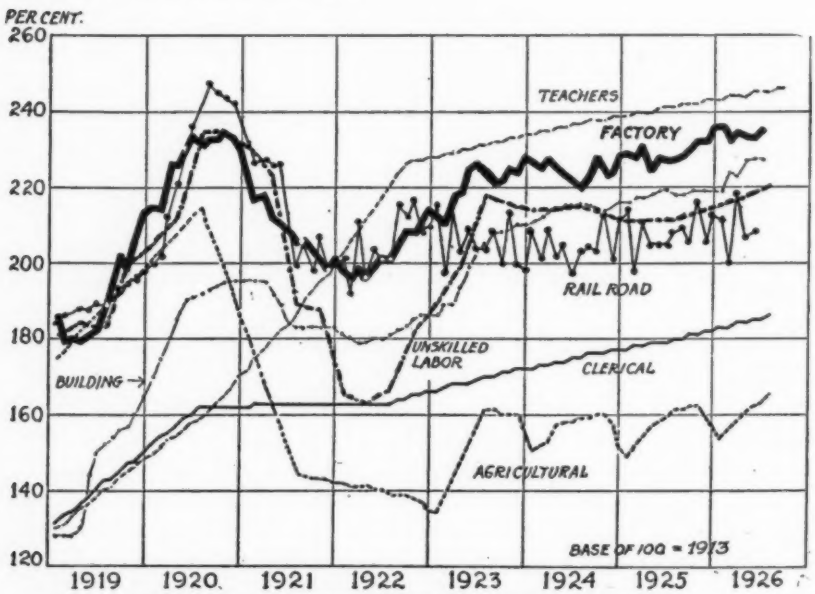


CHART 2. COMPONENTS OF NEW INDEX OF WAGES

Taken as a whole, the composite shows the average wages for the country at the present time nearly one-third higher than at the close of the War; about 6 per cent above the *average* for 1920; and some 16 per cent above the average of 1922, the post-war low year.

All this brings more sharply than ever into contrast the course of wages and commodity prices, and especially of non-agricultural prices. According to the Bureau of Labor compilation, the average of non-agricultural prices, which includes a large element of manufactured and semi-manufactured goods, is now some 80 points, or 33 per cent, *below* the average for 1920, while the average of wages is some 6 per cent higher than for 1920. Yet, in spite of this extraordinary change, the rate of earnings of industrial corporations, on their gross capital, was in 1925 and for the first half of 1926, 25 per cent higher than the average pre-war rate (1911–13 average) or than in either 1923 or 1924—probably higher, other compilations suggest, than in any peace year since the great boom of 1905–06.

This is an astonishing paradox; higher wages, apparently lower prices, and abnormally high profits. It undoubtedly pictures one of the most interesting periods in American industrial history. Even after allowing for the fact that the price index and the wage index may not be strictly comparable, in that the wage index is heavily weighted with factory earnings, it seems clear that such a result could only have been attained by a considerable gain in actual man-product in industry (the phrase "increased efficiency" suggests a gain in the actual efficiency of the workers, whereas a variety of indications suggest that this has really been slight; the gain has been largely in improved factory equipment and factory management).

But the gain to the worker has, of course, been immense—the relative "spread" between the wage level and the average "cost of living" having been in the last three years probably greater than at any time in the last half century. The difference between these three years and the years immediately preceding the War is not less than 20 or 25 per cent. This must mean, for the 30 odd million wage workers of the country, a difference in extra spending power of not less than 6 or 8 billions annually; which, in the writer's mind, goes far to explain the prolonged building boom and huge sales of automobiles and other things formerly classed as luxuries.

In preparation is a new index of property values (securities and real property) and when this has been completed the two new indexes will be included in a revision, from 1913 on, of our Index of the General Price Level. The data and index numbers of the new wage composite follow:

DATA OF COMPONENTS OF NEW COMPOSITE INDEX OF WAGES

Type	Available from	Sources
Agricultural	1922 quarterly 1910 annually 1866 periodically	United States Department of Agriculture, "Crops and Markets"
Building	1913 annually 1921 monthly	Federal Reserve Bank, New York, Reports Department from United States Bureau of Labor Statistics and National Association of Builders Exchanges
Clerical	1914 annually, usually October	New York State Department of Labor, Industrial Bulletin
Factory	1914 monthly	New York State Department of Labor, Industrial Bulletin
Railroad	1921 monthly 1910 annually 1895 every five years	Interstate Commerce Commission Railway Statistics, 1924
Teachers	1913 annually	United States Department of Education and National Education Association Research Bulletin, March, 1925
Unskilled	1914–1924 quarterly 1925 semiannually	Federal Reserve Bank, New York, schedules to about 20 firms in Second Federal Reserve District

A NEW WEEKLY INDEX OF COMMODITY PRICES

By D. W. Ellsworth and Ruth Houghton

This article describes the construction of a new weekly index of wholesale commodity prices. The index as computed at present covers the period from January 6, 1925, to date; but it has been linked with the United States Bureau of Labor Statistics Monthly Wholesale Commodity Price Index for January, 1925, and the new index therefore records the weekly movements of wholesale prices relative to average prices in 1913. The new index is published each Friday in *The Annalist*, giving the average commodity price level, compared with the 1913 base, for the previous Tuesday.

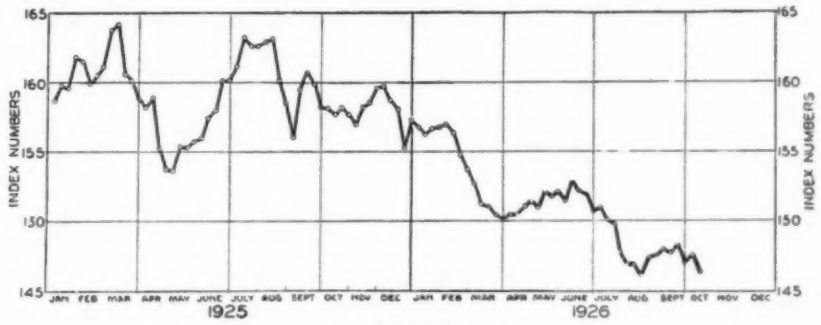


Chart I

The New Weekly Index of Wholesale Commodity Prices
1913 = 100

Chart I shows the weekly movements of the new index, based on Tuesday prices, over the period for which it has been computed. The eight main commodity groups of which the total is composed are: farm products, food products, textile products, fuels, metals, building materials, chemicals, and miscellaneous commodities. Each of these group indices has likewise been linked with the corresponding group index of the United States Bureau of Labor Statistics Wholesale Commodity Price Index for January, 1925, and provides an accurate weekly record, on the 1913 average base, of the average price movements of each of these homogeneous groups.

It is believed that the new index, for the first time in this country, at least, meets fully the requirements of a thoroughly "good" index

of wholesale commodity prices. These requirements may be briefly summarized as follows:

1. Accurate price quotations
2. Representative list of commodities
3. Correct weighting
4. Prompt publication
5. Exclusion of commodities for which there is no established, recognized market for specific grades
6. Division into main commodity groups
7. Publication of full details of construction

Of all the indices of wholesale commodity prices now in existence in this country, that of the United States Bureau of Labor Statistics most nearly meets the requirements listed above. It is based on more than 400 price quotations taken from widely distributed markets for both raw and finished materials and is divided into the following main groups, each of which is in turn subdivided into smaller groups: farm products, foods, cloths and clothing, fuel and lighting, metals and metal products, building materials, chemicals and drugs, house furnishing goods, and miscellaneous commodities. The currently published index numbers cover the main groups, the subgroups and the individual commodities.

In combining the separate price series into group and total composites, the quantity of each commodity marketed in the year 1919 is used as a weight, and a total money sum is secured by adding the total cost of these quantities at their average prices during the month. The total cost of this "bill of goods" is then expressed as a relative to, or a percentage of, the average 1913 cost. A complete explanation of the present method of constructing the index is given in Bulletin No. 367, Wholesale Price Series, published in January, 1925, by the Bureau of Labor Statistics.

A minor defect of this index is the inclusion of prices of certain commodities which obviously cannot be accurately graded for trading purposes. The consequent difficulty of obtaining homogeneous price quotations, for example, on such articles as dining room buffets, davenports or bedroom rockers is apparent. This difficulty is probably overcome to a great extent, however, by scrupulous care in securing quotations on such articles; and the importance of this defect is further minimized by the fact that this type of commodity is given but little weight in the composite.

The most serious defect in the Bureau of Labor Statistics index is the tardiness with which it is published, the index numbers for each

month usually not being published until after the 15th of the following month.

The new weekly index of wholesale commodity prices is based on 74 series of price quotations. Fourteen of these series, moreover, are themselves composites, so that the total number of commodities represented by the index is much more than 74. The following tabulation shows the number of price series in each group and also the number of these series which are themselves composite prices:

Group	Number of series	Number of composites
I. Farm products	17	1
II. Food products	31	0
III. Textile products	5	5
IV. Fuels	6	1
V. Metals	9	2
VI. Building materials	4	4
VII. Chemicals	1	1
VIII. Miscellaneous	6	0
Total	79	14
Duplications	5	0
Net total	74	14

By far the greater number of quotations are taken from *The Journal of Commerce* (New York). In every case where suitable composite prices are available, use is made of them. Wool, cotton cloth, cotton yarn, worsted yarn, raw silk, and rayon prices are represented by the corresponding Fairchild Textile Index Numbers, compiled by the Fairchild Analytical Bureau and published currently in *The Daily News Record* (New York). These index numbers record accurately the weekly movement of textile prices and are used by permission of the Fairchild News Service.

Other published price composites of which use is made in the new index are the *Coal Age* index of spot bituminous coal prices, the *Iron Age* composite prices of pig iron and of finished steel, the *American Contractor* composite prices of lumber, brick, cement and structural steel, and the *Chemical and Metallurgical Engineering Journal* weighted index of chemical prices. Acknowledgment is also made of the assistance of an executive of Armour & Co., Chicago, who through R. D. MacManus of the Department of Public Relations furnished valuable advice on the selection of representative quotations on meat products.

The price series selected represent the most important commodities included in the United States Bureau of Labor Statistics Wholesale Commodity Price Index as given on pages 208–215 of Bulletin No. 367. Several commodities, such as boots and shoes and one main group,

house furnishing goods, are excluded on account of the difficulty (if not the impossibility) of securing homogeneous price quotations. One commodity not represented in the Bureau of Labor Statistics index, namely, rayon, is included in the new index.

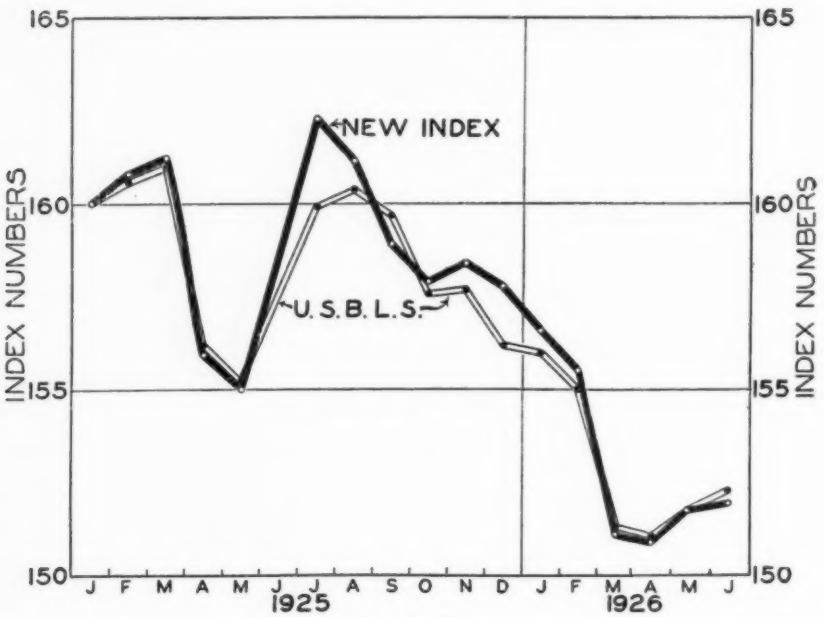


CHART II

THE NEW INDEX OF WHOLESALE COMMODITY PRICES COMPARED WITH THE UNITED STATES BUREAU OF LABOR STATISTICS INDEX

The system of weighting is that used by the United States Bureau of Labor Statistics Wholesale Commodity Price Index, as explained in a previous paragraph. The actual weights used are taken from pages 200–206 of Bulletin No. 367, with adjustments wherever necessary to allow for the fact that some of the quotations used in the new index are for different grades of commodities or are different in number from those used in the Bureau of Labor Statistics Index. At the prevailing prices on March 31, 1926, the relative importance of the eight commodity groups in the new index is as follows:

Group	Per cent
I. Farm products	32.8
II. Food products	26.3
III. Textile products	5.8
IV. Fuels	14.7
V. Metals	9.3
VI. Building materials	4.5
VII. Chemicals	1.8
VIII. Miscellaneous	4.8
Total	100.0

The use of quotations on the most important commodities included in the United States Bureau of Labor Statistics Wholesale Commodity Price Index and the use of the same weighting system result in the close agreement of the movement of the new index with that of the Bureau. Chart II shows for the period over which the new index has been computed the monthly averages of the new weekly index, compared with the monthly index of the Bureau of Labor Statistics. The latter is generally (and rightly) considered the most reliable index of the movement of commodity prices in the United States; and the close agreement shown by Chart II seems to justify the expectation that, with minor variations, the new weekly index will in the future faithfully anticipate the movements of the Bureau of Labor Statistics monthly index.

THE DISAPPEARING "PUBLIC"

With the rise of class alignments, especially along industrial lines, it has become customary to refer to that group which is not directly aligned with any of the recognized self-conscious classes as the "public." Our political system is built largely on the assumption that this un-aligned group, this "public," includes the great majority and that this group will always serve as a control of, or balance to, the selfish programs of any class. In fact, the existence of such a group is an essential assumption in the logic of present political theory and practice. That the facts corresponded with this theory at the time our political foundations were laid and for a long time subsequently, is undoubtedly true. Whether they do so to-day, however, is another question. Such developments as Sovietism and Guild Socialism are a complete abandonment of the idea of a large group which is neutral in industrial controversies. What are the trends in our own country?

As an approach to this question, Professor Hansen some years ago undertook to classify the population of the United States into eight economic classes.[1] Taking into consideration psychological as well as occupational considerations, he grouped these classes into four categories, namely: capital, labor, the public, and unclassified, on the basis of their probable alignment in the class struggle and on political questions. Subsequently this study was duplicated by Phelps for the state of Rhode Island, which, as one of the older and more highly developed states, might be expected to exhibit in a more pronounced form the tendencies apparent for the country as a whole.[2] The present study makes the same analysis of trends in industrial class alignments in the state of Washington, a comparatively new and industrially undeveloped state in the far west. The occupational and industrial trends evident from the three studies are presented in Tables I and II.

TABLE I

PERCENTAGE OF THE POPULATION IN DIFFERENT ECONOMIC CLASSES IN THE UNITED STATES, RHODE ISLAND, AND WASHINGTON, 1870–1920 (U. S. CENSUS REPORTS IN OCCUPATIONS)

Classes[1]	United States[2]						Rhode Island[3]						Washington[4]				
	1870	1880	1890	1900	1910	1920	1870	1880	1890	1900	1910	1920	1880	1890	1900	1910	1920
Farm laborers	23.1	19.1	13.2	15.2	16.1	10.0	6.5	3.3	3.1	2.8	2.6	1.4	10.1	5.0	7.3	7.4	5.5
Farmers	24.0	24.6	23.6	19.8	16.3	15.5	7.3	6.0	4.3	2.8	1.7	1.2	31.2	17.8	14.7	10.5	11.8
Proprietors and officials	4.6	4.6	5.9	6.2	7.5	7.6	6.3	6.6	7.9	7.3	7.9	6.5	4.6	8.9	14.7	9.2	8.4
Professional	3.3	3.8	4.9	5.4	5.4	6.6	3.4	4.0	4.7	6.2	6.9	7.7	7.0	6.1	6.4	7.4	8.2
Low Salaried	2.5	3.0	4.3	4.6	6.3	9.6	4.2	4.5	7.0	9.4	12.0	15.0	1.3	4.9	6.2	10.2	14.6
Servants	7.8	6.2	6.4	5 0	4.1	3.1	7.0	6.2	6.7	5.2	4.7	3.3	2.6	5.0	4.1	3.5	2.7
Industrial wage earners	26.6	30.4	32.4	35.3	38.2	42.4	47.7	51.7	59.3	58.9	57.3	59.5	6.4	6.1	17.3	20.9	23.3
Unclassified	8.1	8.2	9.3	8.5	6.0	5.1	17.6	17.7	7.0	7.4	6.9	5.4	36.8	46.2	29.2	30.9	25.5

[1] For the specific occupations included in each of these classes, see Hansen, *loc. cit.*, Vol. XVII, p. 420 *et seq.*
[2] The figures for the United States are taken from Hansen, *loc. cit.*
[3] The figures for Rhode Island are taken from Phelps, *loc. cit.*
[4] Washington had not yet become a state in 1870.

[1] Alvin H. Hansen, "Industrial Class Alignments in the United States," this JOURNAL, Vol. XVII, p. 417, and Vol. XVIII, p. 503.

[2] Harold A. Phelps, "Significance of the Economic Classes in Rhode Island," *Social Forces*, March, 1926, p. 620.

Table I shows (1) that the same trends in occupational classification are evident in Rhode Island, Washington, and the United States as a whole, although they are most pronounced in Rhode Island, and least pronounced in Washington, (the large percentages of unclassified in this state are partly due to difficulties of classifying new occupations under the categories of the earlier censuses); (2) that the percentage of farmers, farm laborers, servants, and unclassified has been declining throughout the period; and (3) that the percentage of proprietors and officials, professional, low-salaried, and industrial wage earners has been, on the whole, increasing.

Adopting Professor Hansen's method of grouping these occupational classes into the larger categories of capital, labor, and the public,[1] we find again that the same tendencies are present in the three studies: (1) the alignments of capital and labor tend to include an ever increasing proportion of the whole population; (2) this tendency is much more pronounced in an old, developed state like Rhode Island than in a comparatively new and undeveloped state like Washington; and, (3) as a result of the tendency indicated in (1), there is a constant decrease in the proportion of the total population classifiable as "the public" and "unclassified."

The significance of these trends from the standpoint of political theory and practice, the class struggle, and methods of social organization and control is obvious. If it can be demonstrated, as seems clear from the above studies, that the population of the United States is becoming increasingly stratified into class-conscious economic groups, it becomes increasingly important to devise ways and means of social and political organization which recognize these alignments as a modification of, or as a substitute for, the present organization based on territorial groupings.

TABLE II

PERCENTAGE OF THE POPULATION CLASSIFIABLE AS CAPITAL, LABOR, AND PUBLIC

	United States[1]				Rhode Island[2]				Washington			
	Capital	Labor	Public	Unclassified	Capital	Labor	Public	Unclassified	Capital	Labor	Public	Unclassified
1870....	7.1	26.6	58.2	8.1	10.5	47.7	24.2	17.6	....	....	....	....
1880....	7.7	30.4	53.7	8.2	11.1	51.7	19.5	17.7	5.9	6.4	50.9	36.8
1890....	10.2	32.4	48.1	9.3	14.9	59.3	18.8	7.0	13.8	6.1	33.9	46.2
1900....	10.8	35.3	45.4	8.5	16.7	58.9	17.0	7.4	20.8	17.4	32.6	29.2
1910....	13.8	38.2	41.9	6.0	19.9	57.3	15.9	6.9	19.4	20.9	28.8	30.9
1920....	17.2	42.4	35.2	5.1	21.5	59.5	13.6	5.4	23.0	23.2	28.3	25.5

[1] Taken from Hansen, *loc. cit.*
[2] Taken from Phelps, *loc. cit.*

GEORGE A. LUNDBERG[2]

[1] This classification is as follows: Capital—proprietors and officials and the low-salaried; labor—the industrial wage earners; the public—farmers, farm laborers, professional, and servants. *Op. cit.*, Vol. XVII, p. 423.

[2] Assisted by Lydia A. Woodbury.

UNITED STATES CARS IN CANADA

The north-country states in New England have been following the practice for the past two years of making traffic censuses in order to determine the volume of traffic and the proportion of passenger cars and trucks which pass over the various highways. From the resulting data naturally is deduced some valuable information for the Highway Department, throwing light upon the use of the roads both in amount of traffic and the character of it.

A similar traffic census has been taken in the Province of Quebec in 1925 and again in 1926. This census is principally interesting to people in the United States as indicating the proportion of American traffic over Quebec roads. It appears from the census return that in 1926 the survey included 42 main highways and 8 market roads, having a total length of 3,250 miles, of which almost all were of course main roads of the Province. The daily average of vehicles passing over the 42 main highways taken together was 831 vehicles of all descriptions as compared with 722 in 1925, representing a gross increase of 15.1 per cent. Of this total daily traffic, trucks, busses and motor-cycles and horse-drawn vehicles numbered 192, the remainder of the total being touring cars. These were distributed as follows: Quebec, 430; other provinces, 81; United States, 128. Of the total passenger car traffic, therefore, the United States contributed about one-fifth, or considerably more than all the provinces of Canada exclusive of Quebec.

The influx of American cars to Canada during the past summer is more clearly indicated, however, by noting the traffic on those important highways which either lead into Canada or between Montreal, Quebec, and Toronto. The following table indicates the great volume of American traffic very clearly:

DAILY AVERAGE OF TOURING CARS PASSING OVER SIX PRINCIPAL QUEBEC HIGHWAYS FOR SEVEN DAYS (AUGUST 1-7, 1926)

Highway	Quebec Cars	Other Province Cars	United States Cars
Montreal-Sherbrooke	873	23	172
Montreal-Quebec	1,021	61	492
Edward the VII	761	25	990
Sherbrooke-Derby Line	418	3	377
Montreal-Toronto	1,791	347	367
Montreal-Malone	623	24	324
	5,487	483	2,722

The totals being totals of averages, are of course not entirely accurate, but assuming that they are approximate, of the 8,692 cars which traveled in seven days over the six most important arteries in the Province, almost a third bore United States registration plates. The American cars were approximately half as many as the Quebec cars, and were more than five times as many as cars from all the other provinces. It is amusing to observe that on the King Edward the VII Highway the average of United States cars was almost one thousand per day, or a much higher total than all cars combined bearing Canadian plates.

While the motor traffic of citizens of the United States to Canada is naturally large upon its own merit as a delightful method of spending a vacation, the average citizen would naturally jump to the conclusion that the immense influx of American cars indicated by these road figures may be inspired to a large degree by the fact that travel in Canada is accompanied by the comforting thought that thirst may be quenched in a manner especially agreeable to the tourist.

W. S. ROSSITER

PROGRESS OF WORK IN THE CENSUS BUREAU

CENSUS OF AGRICULTURE

All but five or six of the state bulletins presenting the statistics by counties have been published, or are in the hands of the printer. The entire series will contain approximately 2,100 pages. A bulletin summarizing these data for the United States, by states, will be issued shortly. The final report presenting the statistics by subjects and by states but not by counties will be ready for publication next spring.

BIENNIAL CENSUS OF MANUFACTURES

The final and complete report of the 1923 census of manufactures, a volume of 1,450 octavo pages, was published in October.

By the first of December, 160 preliminary reports relating to the census for 1925 had been prepared for publication in the form of mimeographed press summaries. The publication of some of the industry reports is still being held up by the failure of manufacturers to reply promptly to the Bureau's requests for information needed to complete their returns. Of the reports now in preparation, a number could be completed and issued immediately were it not for the dilatoriness of one or two important establishments in each of the industries concerned. This condition has entailed a vast amount of correspondence. During the period from January 1 to November 30, 1926, over 100,000 letters were written in regard to the receipt and correction of schedules.

SURVEY OF CURRENT BUSINESS

The growing interest in current business statistics and the resulting increased output of such statistics are well illustrated by the number of new items recently added to the Survey of Current Business. Taking the August issue, which contained the semiannual detailed tables, 106 new items were shown, as compared with the previous issue. The total number of items now covered by this publication is 1,698.

The new data, added to the Survey of Current Business during the six-month period, present figures on the following activities: Cotton-cloth production, stocks and unfilled orders by groups; trackwork production; cast-iron boiler and radiator data; household enamelware; closed-automobile relative production; electrical bookings; electric hoists; electrical porcelain; band instruments; vacuum cleaners; sales-books orders; barrel headings; cypress lumber; rotary-cut veneer; household furniture; receipts of milk at Philadelphia and Baltimore; Canadian life-insurance sales; and brokers' loans.

VITAL STATISTICS

The state of Arizona was recently admitted to the death registration area and also to the birth registration area.

The death registration area now includes 41 states and comprises 89.8 per cent of the estimated total population of continental United States, while the birth registration area includes 34 states and 76.4 per cent of the population.

MARRIAGE AND DIVORCE

The annual census of marriage and divorce covering the year 1925 has been completed, and the preliminary totals for the United States were given out in a press summary released for use on November 26. The whole number of marriages performed in 1925, according to these preliminary figures, was 1,181,838, as compared with 1,178,318 marriages in 1924, representing an increase of 0.3 per cent. The number of marriages per 1,000 of the estimated population in 1925 was 10.2, as compared with 10.4 in 1924.

The number of divorces granted in the United States in 1925 was 175,495, as compared with 170,952 in 1924, representing an increase of 2.7 per cent, and a divorce rate per 1,000 estimated population of 1.52 in 1925 as compared with 1.50 in 1924.

The number of marriages performed is now reported to the Bureau of the Census by the State Board of Health, or some other state office, in 27 states, and the information with regard to divorces is likewise reported by state officials in 12 states. In the inquiry covering the year 1926 one additional state, namely Mississippi, will be added to the list of those reporting both marriages and divorces. In the remaining states the information has to be obtained from county officials. As considerable time is consumed in getting the information from so many different sources, it is hoped that more states will soon be added to the list of those now collecting information with regard to marriages and divorces in such fashion that the information can be readily supplied for the Federal inquiry.

The final report on marriage and divorce for 1925 is now in press and it is hoped that it will be ready for distribution at some time in February.

J. A. H.

THE PRICE TREND

THE NEW YORK DINNER MEETING OF THE AMERICAN STATISTICAL ASSOCIATION

A dinner meeting of the American Statistical Association was held on Friday evening, September 24, 1926, at the Aldine Club, 200 Fifth Avenue, New York City. About 175 persons were present. The general topic for discussion was *The Price Trend*, most of the discussion, however, being limited to the following five questions:

Why did prices, measured in gold, double between 1916 and 1920?
Why did they shrink greatly during 1920 and 1921?
What causes determined the extent of the shrinkage?
What forces are controlling the present trend?
What is the outlook for the next decade?

President Leonard P. Ayres introduced Professor Jacob H. Hollander, who acted as Presiding Officer. Professor Hollander called attention to the fact that statistics and economics have long been closely related sciences, it being hard to see which gave rise to the other. At present, of course, statistics is the servant of all the other sciences but is used perhaps more by economists than by any other group.

Professor Hollander felt that it was the duty of both economists and statisticians to be social philosophers as well as technicians and he pointed out a unique way in which he believed it possible for them to be of inestimable service to the world. His thesis was that it had never proved feasible to conduct a war on a large scale without resorting to inflation, for nowhere have the people been willing to stand the gaff of taxation to the necessary extent. Furthermore, no people, if fully informed, would tolerate inflation. Hence, if the public can be educated as to the nature and consequence of inflation, war cannot occur.

The first speaker of the evening was Mr. Henry Hazlitt of the *New York Sun*. He presented in a clear and succinct manner the views of the anti-quantity school of economists concerning the forces governing the price trend. In his opinion, the price level in a country like the United States, in which all money has been kept on a parity with gold, is determined primarily by the commodity value of gold. When gold is abundant, its value in terms of goods is small. When it is scarce, its value becomes large.

Mr. Hazlitt described the rise in prices between 1916 and 1920a s having been due to an increase in demand for commodities, due not only to War needs but also to the purchase by speculators of far more goods than were immediately required. The decline in 1920–21 was due, in his opinion, to the shrinkage in such demand, commonly designated as the consumers' strike. He pointed out that the price level during the period under discussion did not vary in proportion either to the supply of gold or to the supply of bank credit as it would have done if the quantity theorists were right in their assumptions, and presented statistics to prove that, in fact, average wholesale prices declined while the supply of gold, of money, and of bank credit was increasing, and vice versa. He felt the view of the quantity theorists that prices are resultant and not causal factors to be fundamentally wrong, and called attention to the fact that, since, obviously, the cause must precede the effect, the decline of wholesale prices beginning in 1920 was clear proof of the erroneousness of the view that prices were the passive factor, for the price level not only started to fall but moved downward a considerable distance, while the volume of money and credit was still increasing. He showed that the mere fact that the volume of bank deposits tends to be closely correlated with the movements of the price level, is no proof whatever that changes in the amount of deposit currency cause changes in the price level; the truth being merely that, when prices rise, business men must borrow more freely in order to cover the same amount of business transactions and that their borrowing necessarily increases the volume of bank deposits.

Furthermore, prices measured in gold are determined in world markets and it is absurd to suppose that the action of the banks in any one country, as for example the United States, can control prices throughout the world. Mr. Hazlitt felt

that the increasing production of commodities and the growing demand for gold in India and in European countries was likely to result in a gold scarcity and cause a downward drift of prices during the next decade.

The next speaker was Professor Irving Fisher of the Department of Economics of Yale University. Professor Fisher set forth the principles involved in the equation of exchange, which, he showed, might be written in abbreviated form as follows:

$$\frac{\text{Money}}{\text{Price Level}} = \frac{\text{Trade Volume}}{\text{Velocity of Circulation}}.$$

In this form, the term "money" is used to cover not only the ordinary varieties of money, but also bank deposits, subject to check. In Professor Fisher's view, the previous speaker had been engaged chiefly in knocking down straw men—these straw men representing different stages of the quantity theory, long ago cast into the discard, none of them being at present endorsed by any reputable economist.

Professor Fisher called attention to the fact that the researches made under the direction of Carl Snyder of the Federal Reserve Bank of New York have shown that, while the volume of trade in the United States has a marked upward trend and the velocity of circulation apparently has a horizontal trend, the cyclical fluctations of these two quantities are extremely similar. This being true, it follows that the ratio of the volume of the circulating medium to the price level should move along a gradual upward trend corresponding in slope to the trend of the volume of trade. He stated that a study by Professor Holbrook Working, recently published in the *Quarterly Journal of Economics*, substantiates fully the existence of this relationship, for Professor Working's figures show that the ratio of "Money" to the level of wholesale prices did pursue a remarkably smooth trend for the third of a century preceding the great War and, since the War, has been following a course parallel to its previous trend. However, Professor Fisher felt that fairness compelled him to admit that, during the period between 1915 and 1922, there occurred a sharp divergence of the line from the trend—a divergence which calls for explanation. He also conceded that, up to the present time, he has not been able to find an explanation that is entirely adequate. However, he pointed out a number of errors in the figures used by Professor Working, which, if corrected, would go a long ways toward explaining the jog in the trend. These errors may be listed as follows:

1. Quoted wholesale prices frequently do not represent the actual prices at which most transactions occur, as a large proportion of deliveries are made on contracts consummated several months previous. This fact has been emphasized by Dr. Walter R. Ingalls.
2. The average of wholesale prices is not a good criterion of the average price level for all commodities. The divergence between the different types of price indices is shown by the study made by Carl Snyder for the Federal Reserve Bank of New York.
3. The figures used by Professor Working to measure the volume of trade are probably not accurate inasmuch as they necessarily depend primarily

upon production figures rather than upon records of actual exchanges. Much of the production during War time was for export and did not enter into domestic trade.

4. During the War period, a large proportion of trade was on governmental account, many of the payments being made by Treasury Warrant, rather than by bank checks. Furthermore, government deposits are not counted as individual deposits subject to check.

Presumably these are part of the explanations for the departure during the War period of the $\frac{M}{P}$ line from a smooth trend.

Professor Fisher took exception to Mr. Hazlitt's assertion that the price decline in 1920 preceded the falling off in the volume of circulating medium. He contended that, although it must be granted that the raw figures do not show the time sequences mentioned by Mr. Hazlitt, when the data are adjusted for changes in the volume of trade, it is found that the changes in the volume of money preceded the changes in the price level in every instance except one. In nearly every case, it is possible to identify the corresponding fluctations in the two curves. The ordinary lag of prices after money supply is shown to be about six months. Similar studies made in European countries yield corresponding results. In Professor Fisher's view, it is entirely impossible to explain the monetary and price changes prevailing in Europe during and after the great War on the basis of the relative value of gold for use in the arts, as compared to that of other commodities.

Professor Fisher congratulated the authorities controlling our Federal Reserve Banks for having adhered to the policy recommended by the delegates of 31 nations at the Genoa Economic Conference of 1922—the policy consisting primarily in the regulation of the supply of money and circulating credit. He stated that the Bank of England is now following the same path.

The third speaker was Dr. W. Randolph Burgess of the Federal Reserve Bank of New York. His remarks dealt principally with the relation of the Federal Reserve Banks to the price level. He pointed out that there was a wide diversity of opinion as to what the Federal Reserve Banks could do if they so desired to control the movements of the price level. He showed that the fluctuation of prices in Europe and the United States have tended closely to parallel each other during the past century and that, under such circumstances, it is evident that these prices are determined by forces world-wide rather than national. He felt, therefore, that it was rather obvious that the Federal Reserve Banks could do but little in the way of influencing the trend of prices in the world as a whole.

Furthermore, even if the Reserve Banks could stabilize prices, the question would arise as to which kind of prices it would be desirable to use as a criterion of stabilization. He showed by means of charts that the courses of wages, prices of farm products, other commodities at wholesale, and commodities at retail, have pursued widely different courses during the period since 1913. The same might be said of the prices of real estate, house rents, and securities. Which of these index numbers should be followed by the Federal Reserve Bank if it sought to fix the price level?

Dr. Burgess pointed out that one of the chief functions of the Bank was to prevent orgies of speculation in various fields which would be likely soon to lead to various financial disasters. Experience has sometimes proven it necessary to limit the output of credit without regard to the course of the price levels mentioned. While the Federal Reserve Banks are unable, with their present powers, to control the trend of prices in the United States, they may affect to some extent the cyclical movements of prices.

There has been much speculation concerning the effect of the Federal Reserve Banks' "open market operations" (that is, their dealings in securities of the Federal government) on the financial affairs of a nation. Experience has shown that such open market operations commonly show close inverse correlation with the amount of money owed by the member banks to the Federal Reserve system. When securities are purchased by the Federal Reserve Bank, the money obtained by the sellers is directly or indirectly used to pay off the indebtedness of the important banks to the Federal Reserve Bank. When securities are sold, the reverse is true. Dr. Burgess was inclined to agree with Mr. Hazlitt that the growing demand of Europe for gold would tend to cause a downward trend of the price level during the next decade.

The discussion was opened by Professor George F. Warren of Cornell University and, in the main, dealt with the price outlook at the present time. He said that he advised all persons interested in this question to study as closely as possible the situation in South Africa with attention directed especially to the amount of gold in sight in the presently operated fields, the prospects of discovering new fields, and changes in the methods of mining which would affect the cost of producing gold.

Professor Warren next pointed out that periods of inflation have always meant prosperity to the farmers as compared to the city people, and vice versa. He held that this was necessarily true because wages tend to lag behind other prices. Retail prices of city products consist mostly of wages; hence, in a period of deflation, this class of price remains higher than the average level of all prices. Under such circumstances, farm prices must necessarily be below the average level. As long as the price level tends to fall, the farmers may expect hard times.

Professor Warren felt that the present evidence indicated that the course of prices during the next few years would be slightly downward.

The discussion was continued by Professor B. H. Beckhart of Columbia University. He pointed out that the great mass of applicable statistical data developed since the beginning of the War has made it much easier for students of money to come to definite conclusions than was the case at the time when Professors Kemmerer, Fisher, Laughlin, and Hobson first began their investigations. This new evidence has resulted in ironing out many of the differences between the quantity theorists and their opponents.

These developments have also made necessary one important modification in the quantity theory, for the figures have shown that, during periods of rapid inflation, the velocity of circulation does not remain constant, as has usually been assumed to be true, but instead increases vastly. Under such circumstances, prices do not rise in proportion to the growth in the volume of money, but at a

far more rapid rate, continuing their upward rush until collapse is brought about by the "scarcity" of paper money, there being not enough with which to do business. Other necessary modifications of the quantity theory have been made in order to take account of the effects of external price on internal prices and the effect of psychological factors upon demand. With these modifications, the quantity theory is now coming to be more and more widely accepted.

He said that the central bankers, who formerly held that their banks could do nothing in the way of controlling the price level, now tend more and more to attempt to exercise such control. Unless the quantity theory is correct, such attempts appear to have no possibility of success.

Professor Beckhart felt that the real differences between Mr. Hazlitt and the other speakers were after all little more than differences as to the amount of emphasis which should be given to various factors. He agreed with the preceding speakers in the view that, unless some new developments in the methods of producing gold should occur, the chances are that production of goods will outrun the production of gold and that, therefore, the trend of prices will be downward for some years to come. This thesis is based upon the supposition that the world will gradually return to a gold standard. He raised the question, however, whether or not nations will continue to submit to this "golden yoke," as Keynes termed it, and whether managed currencies will not soon replace the gold standard in most nations, making "laissez faire" in currency standards a thing of the past.

The meeting then adjourned.

INCREASING LABOR STABILITY

NEW YORK MEETING OF THE PERSONNEL RESEARCH FEDERATION AND THE AMERICAN STATISTICAL ASSOCIATION

A joint meeting of the Personnel Research Federation and the American Statistical Association was held at the Aldine Club, 500 Fifth Avenue, New York City, on Friday evening, October 8, 1926. About 100 persons were present.

Dr. W. V. Bingham, Director of the Personnel Research Federation, presided. In discussing the afternoon meeting, he said that a point which had not been emphasized by the speakers, but which interested him was the interview held with the employee at the time of his quitting employment. Dr. Bingham felt that this interview must be very carefully conducted if one is really to learn much about the reasons for the employee's departure. The technique of the making of such interviews is therefore worthy of careful study.

The first speaker of the evening was Dr. W. A. Berridge of Brown University and the Metropolitan Life Insurance Company; he gave a summary and brief critique of the papers and discussion offered in the afternoon.

Supplementing the descriptive summary, the following points were made:

1. Practically all the speakers had pictured labor turnover as a managerial cost, largely overlooking it as a source of waste to the workers and to the public.

2. Some turnover is not only unavoidable but extremely desirable, not for the sake of sheer variety so much as to improve "the goodness of fit" between each worker and the specifications called for by his job; some shifting from employer to employer likewise entails general economic advantages by facilitating equilibrium in the labor market.
3. Analysis of labor turnover and analysis of labor stability are not antithetical but complementary. Either one, when fully developed, involves use of essentially the same data, and yields results which can readily be transformed into the results of the other. Their relation is analogous to that between a time series of mortality and a mortality table.
4. No single coefficient should be relied upon as "the" turnover rate. Dr. Brissenden is right in urging that the phrase "turnover rate" be abandoned owing to its heterogeneous composition; there is no valid analogy between labor turnover and stock turnover. In place of "turnover rate" we should adopt a more specific terminology—(a) total separation rate, (b) voluntary quit-rate, (c) discharge rate, (d) layoff rate, and (e) accession rate.
5. Even these definitions are not wholly water-tight; there is some leakage, for example, among the several types of separation. Each case must be handled on its own merits—determining to what extent voluntary quits may be discharges or even layoffs in disguise, how long an absentee is carried as such before being classed as a voluntary quit, how re-hires are taken into account, and so on.
6. Although it is axiomatic that voluntary quits are heavy in the first few months of employment, we must not assume that all fluctuation in a time series representing voluntary quit-rates is necessarily due to increase of the accession rate. The cause-effect relationship is, probably, in part reversible; quit-rates being increased by external labor market conditions, and by their increase necessitating added hirings for replacement.
7. The argument that external economic conditions have no effect upon turnover experience, simply because one particular company's voluntary quit-rate fluctuates differently from one particular index of general business conditions, is fallacious. One company may be quite unrepresentative of general turnover experience; furthermore, business conditions form too complex a picture to justify the exclusion of all but one "general" index for consideration. A more comprehensive index of turnover and a less heterogeneous index of economic conditions are necessary to such an argument.

The second speaker was Dr. Max Freyd of the Personnel Research Federation. He pointed out that there are in use two distinct methods of studying business conditions—(1) The statistical method, (2) The experimental method.

The statistical method takes facts as they are and analyzes events as they have actually occurred. It is passive in its nature and is not well adapted to the origination of new policies. It depends for its success on eliminating extraneous variables by means of mathematical adjustments of the data. In this respect

it resembles the procedure following a badly planned experiment, since, in such cases, it is likewise necessary to make just such mathematical adjustments. The statistical method has the advantage, however, of being cheaper and requiring less initiative on the part of the management.

From the efficiency standpoint, the experimental method is much superior to the statistical. The statistical method necessarily deals with conditions under which all the factors may be varying. In the experimental method an attempt is made to control all the factors except one, and then to study the results of varying that factor. While it requires skill to plan an experiment of this nature, experience shows that such experimentation is perfectly feasible, provided that the coöperation of the management can be secured.

Dr. Freyd cited as illustrations of the contrast in method two inquiries in the field of advertising. In the first inquiry a statistical study was made of the keyed returns of different types of campaigns conducted by an advertising agency. This method required involved mathematical treatment of the data, and the lack of consistency in the procedure followed in the different campaigns imposed serious limitations on the results.

The second inquiry was conducted through an experimental study of the effect of a two weeks' window display in 56 drug stores. The sales of the product displayed were checked for each of the four weeks before the display, during the period of the display, and for each of the four weeks thereafter. From these records a curve was constructed showing the effect of the display on the sales of the given article in these stores. This study had two merits.

1. By controlling the experiment, the interference of extraneous forces was reduced to a minimum.
2. The mathematical work on the data was only about five per cent of that required for the statistical study in the first inquiry.

The third speaker was Miss Anne Bezanson of the University of Pennsylvania. She began by defining labor turnover as the

$$\frac{\text{Number of separations}}{\text{Number on the payroll}}$$

She next presented a series of charts showing the results of her studies of turnover in metal working plants in New York, Milwaukee, Philadelphia, and Detroit. Her investigations showed that, when a comparison of the different cities is made on the basis of the total turnover in all plants, the curves for the different cities show only a rough correlation. She demonstrated, however, by other charts that the discrepancies between the different cities are due not so much to differences in business or labor conditions as to the fact that the composition of the metal working industry is not the same in the different cities. Labor turnover is much higher in the heavy pressed-metal group than in the other metal working fields; hence, where plants of the former type are abundant, a high turnover rate is found. She further illustrated by graphs the fact that, in the four cities, the turnover rates in the heavy pressed-metal group fluctuated almost identically,

showing that, as a matter of fact, this branch of the metal industry is dependent upon general industrial conditions and that these industrial conditions are approximately the same in different parts of the United States. Evidently then, labor turnover is affected materially by what goes on in other plants as well as by the internal conditions in the plant itself.

From the standpoint of the employer, Miss Bezanson feels that it is more important to study the trend of the turnover than its actual volume. She pointed out that important additional light could be thrown upon the problem by ascertaining the trend of the percentages of "help wanted" advertisements devoted to each class of help. Her charts indicated that there is a marked connection between changes in the volume of such advertisements and the movements in labor turnover. She stated that recently in Philadelphia there has been a sharp rise in the percentage of advertisements calling for skilled labor.

The first participant in the discussion was Professor C. S. Yoakum of the University of Michigan. He expressed the view that we have not yet discovered what lies back of labor turnover. The employee is interviewed when he is hired and sometimes is interviewed when he leaves, but no one knows what is going on in his mind during the interim between these interviews. To understand labor turnover thoroughly, it is essential to study the psychology of the employee during the period mentioned. He felt also that students of labor problems, in addition to considering the relationship between the laborer's attitude and the turnover rate, should analyze in detail the connections existing between the labor turnover in a concern and the production, and other factors measuring the success of that particular business enterprise. This procedure might yield results more valuable than those obtainable through study of relationship of labor turnover to business conditions in general.

The discussion was continued by Mr. Henry S. Dennison of the Dennison Manufacturing Company. He stated that manufacturers first began to take interest in the questions of labor turnover when the fact was called to their attention that the hiring and training of new employees is an expensive process. The older view was that, as long as new men could be readily found to replace the old, it made no difference how many quit. Mr. Dennison emphasized the fact that manufacturers are not interested in the precision of measurement of labor turnover but that they are much interested in methods of lessening its cost in their particular plants. Scientists, however, must never relax their attempts to gain ever greater precision. He felt that the fact, to which Miss Bezanson had called attention, namely that the manufacturers whose records she had obtained were anxious to be placed in categories which would make their turnover rates appear relatively low, is a most encouraging symptom, for it shows that manufacturers are really becoming socially minded—are realizing that they have obligations to their employees as well as to their stockholders. The speaker heartily endorsed the view of a recent writer who pointed out the great value of "outrageous hypotheses." Mr. Dennison felt that the making of uncommon assumptions furnishes a great stimulus to fresh thinking and might well be practiced occasionally by all engaged in the field of business.

The discussion was closed by Professor Sumner H. Slichter of Cornell Uni-

versity. Professor Slichter stated that the program was all directed to answering three questions.

1. Why shall indices of labor turnover be constructed?
2. How shall such indices be interpreted?
3. How can practical use be made of these indices?

Professor Slichter criticised the presentation of figures lumping together all classes of labor turnover. Such a procedure, he said, is illogical because the different types of turnover are, in many respects, opposite in their nature. Discharges are the antitheses of resignations. Both of these, however, are related to conditions in the plant and the personal characteristics of the employees and those under whom they work. On the other hand, lay-offs are radically different in their origin. Their prevalence depends not at all upon personal considerations, and commonly they are almost unrelated to conditions in the plant. Instead, they are as a rule, dependent primarily upon the fluctuations occurring in general business conditions. If then, we hope to succeed in using the statistics of labor turnover to show what conditions really exist and what remedial measures, if any, are required, it is obvious that the three different types of turnover must be dealt with separately.

The meeting then adjourned.

MISCELLANEOUS NOTES

Representatives of Women's Trade Unions Endorse the Plan of the Committee on Governmental Labor Statistics of the American Statistical Association.—The following resolution was unanimously adopted at the Tenth Biennial Convention of the National Women's Trade Union League of America, meeting in Kansas City, Missouri, June 28 to July 3, 1926:

WHEREAS, Unemployment due to seasonal fluctuations and to recurrent changes in general business activity affects disastrously the homes and standards of living of wage-earners and is of vital concern to all women, and

WHEREAS, Many industries in which women are largely employed are peculiarly susceptible to seasonal fluctuations, and

WHEREAS, Recent studies of the problems of stabilizing business have shown that current compilation of adequate information about the trend of employment would aid in preventing or at least lessening the severity of these fluctuations, and the advantage of knowing these facts is already demonstrated in the statistics now made available by the Federal Bureau of Labor Statistics and by the State Departments of California, Illinois, Maryland, Massachusetts, New York, Pennsylvania, and Wisconsin, therefore be it

RESOLVED, That as representatives of women in industry, we urge every state department of labor to undertake as soon as possible the monthly collection and prompt publication of statistics of employment and earnings in accordance with the plan recently advocated by the Committee on Governmental Labor Statistics of the American Statistical Association, whereby each state will collect information about the industries within its own borders, to be published separately for the state and to be transmitted to the Federal Bureau of Labor Statistics for use in formulating national indices of employment; and that we respectfully suggest that these facts be collected and tabulated in such a manner as will show the trend of employment and earnings for women workers separately from the facts about men.

United States Bureau of Labor Statistics.—The following field investigations are now being carried on: Outdoor recreation and welfare work for industrial employees; labor productivity in various industries; development of the coöperative movement in the United States; apprenticeship systems in the United States; Mexican colonies in the United States; wages and hours of labor in the iron and steel industry, men's clothing industry, automobile industry, cotton goods industry, woolen goods industry, boot and shoe industry, and hosiery and underwear industry.

As soon as possible after the beginning of 1927 it is planned to publish a yearbook of labor statistics covering the year 1926. This yearbook will constitute a compendium of developments during the year.

New England Council.—The movement now in progress for almost a year in New England to create an All New England Council for the purpose of taking concerted measures for the material advancement of the six states, in the opinion of its promoters is now fully justifying its existence. Systematic effort has been made during this brief period in which the Council has been functioning to accumulate reliable data, statistical and otherwise, concerning various problems which the six states have in common, particularly water-power, transportation, marketing, both of agricultural and manufactured goods, taxation, etc. Quarterly meetings have been held in different states, and fidelity and enthusiasm have marked the activities of the Council and its various committees.

The New Hampshire delegation has led the way by pointing out the possibility of considering internal state problems and making constructive recommendations looking to improvement. In particular, they are concerned with the decrease of rural population in New Hampshire, which is common to all the north-country states, and have submitted to the Governor and thus to the people of the state a series of twelve definite recommendations looking to state betterment. This policy has been commented on by other states most approvingly, and it is likely that a similar procedure will be followed elsewhere.

The next general meeting of the Conference will occur at Hartford on the eighteenth and nineteenth of November. It is likely to be attended by from eight hundred to one thousand delegates from various organizations in the six states, who will then select for the second time the twelve representatives from each state to form the permanent Council, and there is every indication that the second year of the activities of this organization will start with increased strength and the belief that the creation of this movement means much to the six states.

Annual Meeting of the American Statistical Association.—This year the Annual Meeting of the American Statistical Association will be held in conjunction with the meetings of the other Associations at St. Louis, from December 28 to 31 inclusive. The Headquarters will be at the Hotel Statler. The railroad rates will be, as usual, one and one-half fares for the round trip to members of the Association.

A commission appointed by the Canadian Government in May, 1926, to investigate the prevailing economic depression in the Atlantic Maritime Provinces of Canada and the general question of the rights of these provinces under Confederation completed its work and handed in its report during September. As an addendum to this report a special memorandum on the social and economic progress of these provinces since Confederation (1867) has been prepared by the Dominion Bureau of Statistics and will be issued shortly.

Dr. F. M. Schmidt-Ernsthausen, of the Department of Agricultural Economics, Cornell University, Ithaca, New York, was killed in a railroad wreck in Germany on August 18, 1926.

Mr. Percy C. Papps, Mutual Benefit Life Insurance Company, Newark, N. J., died on May 18, 1926.

Dr. G. E. Harmon of the Department of Hygiene and Bacteriology of the School of Medicine of Western Reserve University has returned to Cleveland after a year spent in research and study with Professor Karl Pearson at the Biometric Laboratory, University College, London.

Joseph E. Pogue, Consulting Engineer, spent the month of October in a trip to the oil-fields of Oklahoma and Texas, making a study of the crude oil production outlook.

MEMBERS ADDED SINCE SEPTEMBER, 1926

Achenbach, Moore S., The H. K. McCann Company, San Francisco, Cal.
Agger, Dr. E. E., Rutgers University, New Brunswick, N. J.
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Brunsman, Howard George, Bureau of Business Research, Ohio State University, Columbus, Ohio.
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Crowther, Samuel, Bayside, N. Y.
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Jones, Dr. Thomas E., President of Fisk University, Nashville, Tenn.
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Keller, N. E., Pacific Portland Cement Company, San Francisco, Cal.
Lagerquist, Dr. Walter E., Irving Bank and Trust Company, New York, N. Y.
Lawrence, George J., Detroit Steel Products Company, Detroit, Mich.
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REVIEWS

The Biology of Population Growth, by Raymond Pearl. New York: Alfred A. Knopf. 1925. xiv, 260 pp.

The Natural Increase of Mankind, by J. Shirley Sweeney. Baltimore: The Williams and Wilkins Co. 1926. 185 pp.

Those who are familiar with Dr. Pearl's writings are prepared to find an abundance of ingeniously obtained new data, a fresh and stimulating point of view on old problems, and a highly entertaining style of presentation. It is not unlikely that they are also prepared to find themselves unable to agree with the author at all points. The present book should live up to all expectations.

The growth of human populations is dealt with as comparable to that of organisms. The book begins with a consideration of the growth curves of such things as white rats, pumpkins, and tadpoles' tails. From these the author passes to populations of yeast cells and to an instructive series of experiments of his own on the growth of populations of fruit flies in universes consisting of pint or quart milk bottles. He then deals with the growth of nations, including ones in which conditions are as different as in the United States, France and Algeria, the last of which he deals with in great detail. He shows that in all of these cases there is much the same *S*-shaped curve of increase in size or numbers.

As in previous more technical publications, the author attempts to bring all cases under a general law of growth, expressible in mathematical form. He uses the general logistic curve

$$y = d + \frac{k}{1 + e^{a_1 x + a_2 x^2 \cdots a_n x^n}}$$

in which d and k are the lower and upper limits of the cycle of growth under consideration.

It may perhaps be questioned whether the capacity which this mathematical form undoubtedly possesses for fitting growth and population curves has any very deep significance. The natural law of unchecked growth is of course that the rate of increase at any moment is proportional to the amount of growing substance or number of reproducing individuals, $\left(\frac{dy}{dx} = ay\right)$, leading to the compound interest law, $y = ce^{ax}$, a curve which rises with continuously upward curvature from *zero* as an asymptote. In nature, however, the possibilities of increase are always limited. It is estimated that the progeny of a single oyster have the potentiality of multiplying at such a rate as to make a pile of shells eight times the volume of the earth in four years. Actually there is little danger of this happening. It is perhaps conceivable that an organism or population may continue growth according to the compound interest law until it bumps abruptly into a limiting condition. It is much more likely, however, that there

will be increasing adverse pressure as growth goes on, leading to damping off and reversal of curvature, and ultimately, if conditions are uniform, to an asymptotic approach to an upper limit. In this we have simply described a general *S*-shaped curve, which may take various forms depending on the exact nature of the adverse pressure. Anything growing under constant or even changing conditions, provided the changes are sufficiently gradual, can hardly be expected to grow in any other way. Any flexible mathematical formula which gives this general shape can hardly fail to give an empirical means of fitting such a curve. The logistic curve is, perhaps, the most convenient but it is by no means the only mathematical form with this property.

The logistic curve in its simplest form, $y=\frac{k}{1+e^{a(b-x)}}$ represents a growth process in which the pressure from the limiting conditions is exerted in a very simple way, as may be seen from the differential formula $\frac{dy}{dx}=ay\left(1-\frac{y}{k}\right)$. The growth rate, while tending to increase with increase in the amount of growing substance, y, tends also to vary directly with the closeness of approach to the limit k. The constant b locates the point of origin relative to the cycle, being merely the distance to the point of inflection, which in this simple form of the curve is the point at which growth is just half completed. Populations of fruit flies and certain human populations, notably the United States, follow this simple law of growth very satisfactorily. Indeed, populations from yeast cells to man appear to conform to it much more closely than do individual organisms, which in general show a point of inflection at an earlier stage. Perhaps this points to a fundamental difference in the nature of the limiting conditions. In populations the inherent reproductive capacity of the individuals is not necessarily changed as the cycle advances. The restraint to growth is external and apparently does increase as a certain limit is approached in some conformity with the simple rule described above. In organisms, on the other hand, the damping off of growth depends more on internal changes in the cells themselves, the process which Minot called cytomorphosis. The average growth power as measured by the percentage rate of increase tends to fall at a more or less uniform percentage rate, leading to asymmetrical types of *S*-shaped curves of which the form $\log\log\frac{k}{y}=a\ (b-x)$ is a simple example, instead of the logistic curve, $\log\left(\frac{k}{y}-1\right)=a(b-x)$.

The argument for the significance of the logistic curve as an expression of a natural law of growth of populations is somewhat damaged, however, by the necessity which Pearl finds for superimposing one cycle upon another. With the exception of the United States his human populations rise from lower asymptotes which are not zero. The explanation offered is that at intervals a cultural change, such as a change from an agricultural to an industrial civilization, more or less abruptly raises the upper limit and starts a new cycle. This, of course, must be granted. It is not clear, however, why the new cycle should be super-

imposed upon the old. A priori, one would expect to find instead a more or less abrupt transition to a new *simple* growth curve (lower asymptote zero). If one transfers a few fruit flies from a pint bottle in which the population cycle is well advanced to another bottle, and then deals with the two bottles as a single population, one would expect a composite growth curve, one cycle added to another. If on the other hand the entire population of the first bottle, flies, pupae and larvae, be transferred without disturbance to a quart bottle, one would expect to find the population increasing soon in just the same way as a population which had developed from the beginning in the larger bottle and had reached the same figure as the first population. A composite curve seems logical enough for the British empire but for Germany or Algeria a change from one simple growth curve to another seems more reasonable.

If, nevertheless, it turns out that nations really grow by superposition of cycles, as Pearl describes them, it would tend to indicate that the growth of human populations is not, after all, comparable to that of fruit flies in a bottle. It is possible that the form of the population curve is, in the main, merely a reflection of progress in the ability of man to deal with nature. The development of the industrial applications of a new idea may well be conceived as something superimposed upon the previous state of industry. From the biologic viewpoint, such changes are changes in the upper limit of the population curve. The question is whether we are to consider populations as growing toward more or less remote limits, which change only occasionally, or whether we are to consider them as always close to their upper limits which, however, are themselves continually changing in more or less definitely cyclical fashion. It is not necessary to hold either view to the complete exclusion of the other.

In any case it appears very doubtful whether predictions are justified that are based on extrapolation for more than a few years. It is only fair to say that Dr. Pearl himself does not wish to lay much stress on this side of the matter. The growth curve may be looked upon as of significance rather in furnishing constants such as a, b and k of the preceding formulae, which describe certain momentary properties of the population.

The later chapters of the book are devoted to researches into the nature of the factors which affect population growth and especially those which cause its damping off. The author finds reason for ascribing a major effect to density of population in the case of fruit flies and also in populations of hens. He finds some rather meagre evidence that the same factor is at work with man. The correlation between birth-rate per 1000 married women, 15–44 years of age, and persons per acre in 132 big cities of the United States came out $-.037 \pm .059$. By holding 4 other quantities constant a partial correlation of $-.175 \pm .057$ is reached. Even taking this figure at face value (it might occur about one time in 25 merely by chance) it surely does not mean very much. As the degree of determination is equal to the square of the correlation, it appears that there is room for more than 30 other factors, uncorrelated with density or with each other, yet of equal importance in determining birth-rate.

What the nature of these factors may be is, as the author points out, a fascinating subject for further research. Evidence on the negative correlation

between wealth or social position and birth-rate and its causes, particularly in the relation between social position and sexual habits, are treated in the later chapters. The very definite relations which the author finds in this case only increase the difficulties of the general problem.

Dr. Sweeney also deals with population questions. He uses Pearl's "vital index," which is simply the ratio of births to deaths as a measure of the "present worth of the evolutionary survival value of a population." He has obtained this index for countries in all parts of the world and for as many years as he could find data. The result is a very instructive survey of the world situation with respect to population.

He finds, for example, average indices above 200 per cent only in New Zealand, Australia, Uruguay, Argentine Republic and Canada, aside from certain small island populations. The United States and most of the North European countries show indices well above 150. Japan, European Russia, most of the South European countries, and Algeria and Venezuela are examples of countries between 125 and 150. Chile, British India, France and British Guiana fall between 100 and 125. Mexico, Straits settlements, Singapore and a number of other tropical regions show vital indices below 100. The trend of the vital index is also investigated. It is found that the great majority of countries show rising trends, the most noteworthy exception being France with average index but little above 100 and South Australia and New Zealand with such extraordinary high indices that some decline seems inevitable. It is somewhat surprising to find that there is no significant correlation between magnitude of index and trend ($-.086 \pm .074$). Relatively high negative values are found, however, within natural groupings of countries.

One of the most interesting studies is on the effects of the World War and the following influenza epidemic. The author finds only temporary effects on the vital index, the years 1920 and 1921 showing practically complete return to pre-war levels.

In a general way, the significance of these facts is clear. The precise interpretation to be put upon the birth-death ratio is not so clear. The author speaks of it as measuring natural increase. This it certainly does not do in an exact way. It is true that indices above 100 per cent imply naturally increasing populations and indices below 100 naturally decreasing ones (effects of emigration and immigration not being considered) but two countries with the same birth-death ratio may be growing by natural increase at very different rates, depending on whether both birth-rate and death-rate are high or low. The birth-death ratio for both Austria and Algeria was about 133 during the period of study. The reader is given no means of estimating the relative percentage rates of natural increase (excess of birth-rate over death-rate) in the absence of data on the death-rates or birth-rates by themselves.

The author emphasizes especially the interpretation of birth-death ratio as a measure of the "biologic soundness" of the population. It may be questioned, however, whether this phrase as used means anything more than its definition, birth-death ratio. The reader must continually be on his guard against reading into it or such phrases as "evolutionary survival value" any necessary implica-

tion as to the inherent qualities of the stocks compared. The author is careful to point out that differences in the birth-death ratio may be due to physical environment or to cultural conditions as well as to genetic differences in the populations. He contrasts, for example, the birth-death ratios of the French of France and of Quebec. He points out that immigrants of any stock soon tend to acquire the birth-death ratio of the people among whom they have come to live, as indicative that "biologic soundness" has more to do with external conditions than to heredity.

Although Dr. Sweeney's book comes from Dr. Pearl's laboratory, he makes little attempt to correlate his findings with regard to birth-death ratios with Pearl's growth cycles. In a population following the logistic curve of growth, the percentage rate of growth falls off directly with the approach to the limiting value. The vital index need not follow exactly this course: indeed it is quite possible for it to hold its own or even rise during the middle portion of the cycle, owing to progressive decline in the death-rate. In a general way, nevertheless, high values are to be expected early in the cycle and low values as the cycle approaches completion. Thus Sweeney's "biologic soundness" should mean to some extent the same thing as position in the growth cycle in Pearl's book. From Dr. Sweeney's viewpoint, the tendency toward persistence of a characteristic degree of "biologic soundness" by a population inevitably leads to overpopulation. He holds that "some of our large and aggressive countries are relatively overpopulated today." He sees things tending toward an "absolute overpopulation with Nature alone taking the situation in hand in ways by no means pleasant." He sees no remedy but an "international agreement to control numbers—by a league of stationary populations." Dr. Pearl's phraseology of cycles of growth leads to a more optimistic viewpoint.

> Should the somewhat inexorable advance in the number of people on this globe which the biological law of population growth makes extremely probable, lead us to view-with-alarm? It seems to me that if we so conclude we ought equally to be concerned about the orderly growth of any baby into manhood or womanhood. . . . Will this process necessarily increase the sum total of human misery and wretchedness in the world, as we are told it will? I used to think so but the longer I have pondered over the matter the less sure I feel about this conclusion. . . . Even allowing for the misery entailed by the Great War it is surely true that at no previous time in the recorded history of mankind was so large a proportion of the population on the whole so happy, and so socially and humanly well off as they are right now. France, for example, is a country with a population near the maximum for the present cycle. But he who thinks France an unhappy or wretched country is both ignorant and stupid.

SEWALL WRIGHT

The Tariff on Wool, by Mark A. Smith. (Publication of the Institute of Economics, Washington, D. C.) New York: The Macmillan Company. 1926. 350 pp.

This is a good book and a good specimen of the publications of the Institute of Economics. It gives a compact, well-arranged, accurate, judicial statement of the essentials on an important and disputed subject. It gives the historical

antecedents of the present wool duty; the facts on sources of wool supply, imports, production, consumption; the characteristics of the duty under the tariff act of 1922; the ascertainable effects of the duty as it stands; and the effects which would ensue if it were abolished. All is made easy for use by the cursory reader, by the ready-reference newspaper man, not least by the ordinary member of Congress. There is a summary at the close of each chapter, and a concluding chapter which sums up the whole. Difficult and troublesome matters are relegated to the appendix. In every way the book is an excellent example of the question-of-the-day publication.

The character of the volume, and the wish to make it tell on public opinion, lead sometimes to elaboration of the obvious. An extended examination of the burden of the duty—in dollars and cents, carefully done, and duly recognizing the extreme difficulty of reaching accurate figures—ends with the sort of conclusion that might readily be reached without prolonged study: "The people of the United States are not as well or as cheaply supplied with clothing and other wool manufactures as they would be if no wool duty were levied" (p. 266). And similarly, in the concluding chapter, it is laid down, in italics, that "the wool duty has effects which are burdensome to the consumer of wool goods" (p. 278). On the other hand, the desire to be scrupulously fair to the protectionists leads to concessions that may comfort the enemy even though they give him little aid. "*If* (the italics are mine) the small additional amount of wool produced because of the duty is necessary to enable the country to meet sudden emergencies, the wool tariff is valuable in the scheme of national defense." When it comes to more intricate matters, it must be confessed that the discussion at times leaves something to be wished. The matter of the cost figures on wool, the problem of joint cost for mutton and wool, the real meaning of those instances in which the allocated cost of wool is worked out to be less than nothing—these are not only relegated to the appendix, but even there are not probed deep.

The conclusion of it all is that wool of every grade—not merely the non-competitive carpet wools, but the clothing wools as well—should be admitted free of duty. I do not see how an unbiased and intelligent person can come to any other conclusion. It is refreshing that, with all moderation in statement, with every avoidance of phrases that might irritate the staunch protectionists, the outcome is so unequivocally stated. The volume should have the more effect on public opinion because of the moderate and judicial temper in which the argument is conducted and this final result presented.

The wool duty stands on firmer ground, however, than that of logic or economics. It has the support of tenacious popular fallacies, such as make the farmer ordinarily a loyal protectionist, even an ardent one. It has the support, too, of the manufacturers, who feel that the farmers must be placated, lest they turn after all against the whole protective system. And not least, it has the powerful political backing of the ranching states. Small as is the population of these states, inconsiderable their contribution to the total output of goods, each of them has its two senators, and its apportioned representatives on political committees and party conventions; and between them they have the chance of turning the scale in a presidential election. One may predict—as I am disposed

to do—that sooner or later the duty will go; but who can say just when that blessed day will come, or by what turn of the political wheel the time will be set?

F. W. TAUSSIG

Harvard University

Interest Rates and Stock Speculation, by Richard N. Owens and Charles O. Hardy. New York: The Macmillan Company. 1925. 197 pp.

Interest Rates and Stock Speculation, by Richard N. Owens and Charles O. Hardy, sets out to test by quantitative methods the relationship between movements of short-time interest rates and stock speculation. Six quantitative tests are made, four of which cover general movements of short-time rates and industrial stock prices during the fifty years, 1872–1922, and two of which cover shorter periods in more intensive fashion. As all yield negative results, the authors carry the investigaton farther and in some cases make similar comparisons of movements in short-time interest rates with fluctuations in the volume of stock sales, which also give negative results. On the basis of these investigations and their analysis of the theories behind them, the authors lay down the thesis that interest rates play a very minor part, if any, in determining the level of stock prices.

Although the authors do not make this clear in the text, it is apparent that they are testing two radically different views of the relationship between movements of short-time interest rates and stock prices. Five of the six tests are concerned with the hypothesis that advancing short-time money rates cause stock prices to fall and declining rates cause stock prices to rise. The other test, however, undertakes to determine whether low short-time money rates result in rising stock prices and high short-time rates in falling stock prices. The five tests covering the first hypothesis would give positive results if the course of stock prices, either immediately or after a regular time interval, moved in the opposite direction to short-time money rates, while the sixth test would give a positive result if stock prices rose while short-time interest rates continued low, and fell during periods when short-time interest rates were high.

The study has tested the first hypothesis exhaustively and has probably disposed of it for all time. First, day-to-day changes in call loan rates are compared with day-to-day fluctuations in both stock prices and the volume of stock sales for a period of thirty-two months which include a bull market, a bear market, and an irregular market. Second, using average monthly figures, several unsuccessful attempts are made to establish a seasonal movement in stock prices and the volume of stock sales to be compared with seasonal movements in interest rates. Third, on a monthly basis for the fifty years, 1872–1922, rising stock prices are compared with falling rates on open market commercial paper, adjusted for seasonal variation, and falling stock prices with rising rates on the same paper. Fourth, open market rates on commerical paper are correlated with security prices over the fifty years after both have been adjusted for trend. Fifth, for the period 1920–1924, interest rate movements are compared with fluctuations in the

prices of preferred stocks, dividend paying common stocks, and non-dividend paying common stocks. The mere enumeration of these five tests is sufficient to illustrate the comprehensive manner in which this hypothesis is tested. Since the results of the five tests are in the main negative to the hypothesis, it is doubtful whether the conclusion of the authors that movements of short-time money rates do not of themselves cause an opposite movement in stock prices either immediately or after a regular time interval, can be successfully refuted.

As a matter of fact the theory back of the remaining test in which low short-time interest rates are compared with rising stock prices and high rates with falling stock prices would lead one to expect the negative results established by the five tests described above. This theory does not state that stock prices, to the extent to which they are influenced by money rates, vary inversely with movements in short-time rates, but that the stock market, being a part of the market for long-time capital funds, is influenced by interest rates among other things. It holds that the long-time and short-time markets for funds are to a certain extent competitive and that in times of wide spread between the price of money in the two markets, there may be expected a shifting of funds from one to the other which will find its expression in the prices of bonds and stocks. Stated in its most simple and, therefore, probably most unrealistic form, this theory holds that when funds invested in short-time loans render a materially lower return than funds invested in stocks and bonds, there is a tendency toward higher security prices because lenders and investors of money withdraw their funds from short-time loans, and speculators borrow short-time funds, all in order to invest in securities. Conversely, when the return on short-time loans is materially higher than the yield on securities, there is a movement of funds in the opposite direction which tends to lower security prices. Now it is obvious that this theory has not been tested in the five tests described above. In none of those tests, nor in fact, anywhere in the study, is the long-time interest rate, or yield on securities, ever taken into consideration. Those five tests were essentially designed to find out whether, when call rates, for example, rose from 2 to 3 per cent, stocks fell in price, and when call rates dropped from 10 to 9 per cent, stocks rose. The five tests showed that stocks in general did not act in this manner and that is exactly what would be expected on the basis of the theory that the long-land short-time markets for funds are related and find an expression in stock values. Under this theory, for example, if the long-time interest rate, or yield on securities, averaged around 5 per cent and call rates were 2 per cent, one would expect banks and other investors to buy securities in greater volume and security prices to rise. When call rates rose from 2 to 3 per cent, they would still be lower than the long-time interest rate and one would expect security values, not to fall, but to continue to rise. Conversely, if call rates were 10 per cent, one would expect security values to fall and this would still be true after call rates had dropped to 9 per cent. Under this theory it is only when short-time rates cross long-time rates that a reversal of security values is to be expected. The actual market operations to substantiate this theory have never been adequately described, but in general it rests upon observed phenomena. It has seemed to explain movements in bond prices and, therefore, to a considerable extent in stock prices, as Colonel Leonard Ayres has

shown a high degree of correspondence over long periods between fluctuations in prices for stocks and bonds. Statistics of the growth of bank investments in recent years have also agreed with the main features of the theory.

The study pretends to test this theory by examining the movement of stock prices during periods of low and of high rates on call loans and open market commercial paper. The test is rendered worthless, however, by the definition employed in determining what interest rates will be considered high and what will be considered low. During the fifty years covered by the test, there are periods in which bond yields averaged around 6 per cent and others when yields were less than 4. A rate on call loans of 5 per cent, therefore, would be low in relation to long-time rates in the first case and high in the second, and one would expect a test of rising or falling stock prices relative to low or high rates on short-time loans definitely to take account of changes of this sort. In the test in this study, however, no allowance is made for such changes, and rates for call loans and open market commercial paper are defined as high or low according to whether they are above or below their own median for the fifty years. Such a definition obviously invalidates the results of this part of the study. As an illustration of the position to which it commits the authors, it is sufficient to record that call rates are high at the present time in terms of this definition and have been so since November, 1924. Now according to general financial and banking opinion, call rates were low during much of 1922 and during most of the period since early in 1924. Accompanying these low rates, prices for stocks and bonds have been generally upward. In only six months of this period, however, have call rates averaged below 3.31 per cent, their median for the period 1872–1922, and in only those six months, therefore, would they be considered low in the study. If the comparison of low call loan rates with stock prices had been carried through the first half of 1926, therefore, the great majority of months of rising stock prices in 1922 and in the period from November, 1924, to the present would not be considered as confirming the theory but as refuting it. With such a definition of high and low rates it is obvious that the negative results obtained in the study are in no sense conclusive as applied to the theory in question.

In the statement of the problem which occurs in the first chapter of the book there is no clear distinction drawn between the two theories of the relationship between interest rates and stock prices that are subsequently tested. By far the greater emphasis is laid upon the second theory, however, and the discussion turns largely around the speculator who buys because he can borrow money at a cheaper rate than the return on the stocks which he purchases. This is particularly unfortunate in view of the fact that five of the six tests, and the five in which the authors score their most impressive results, are designed to test an entirely different sort of relationship between movements in short-time money rates and stock prices. It would be illuminating to the reader to know just what wide body of informed opinion has subscribed to this now-discredited view. As the volume stands, only one of the six tests, and that questionable on statistical grounds, really meets the theory that has been most widely held and that forms the subject of most of the introductory discussion in the text.

The introduction can also be fairly charged with over-stating the case which it

sets out to disprove. "One of the most universally accepted principles of finance," states the preface, "is that stock market speculation is governed primarily by variations in interest rates." Now in view of the great attention paid by both the financial community and economists to other factors affecting stock prices, such as business conditions, present and prospective earnings, stock dividends, pool activities, bear raids, etc., with which financial literature relating to the stock market is filled, the assertion that stocks are supposed to respond primarily to movements in interest rates greatly overstates the theory to be discussed. Nor is this overstatement substantiated in the long list of quotations given in support of the text. Many of the writers quoted undoubtedly held the view that interest rates were a relatively important element in the stock market, but rarely went so far as to make them the only one. While such overstatement is relatively immaterial in the discussion of the theory, it becomes of vital importance in the quantitative tests that are made, because nowhere in these tests is allowance made for other factors which might influence stock prices. The result of this is that the series of negative results, impressive as they are in five of the six cases, are inconclusive. There still remains the possibility that interest rates are the chief influence affecting stock prices but that their bearing is obscured by the cumulative effect of other factors, individually of smaller importance.

The general public of business men and speculators, however, will probably be more interested in the conclusion of the authors than in either the nature of the statistical methods used or the hypotheses to which they are applied. These conclusions may best be summarized by direct quotation from the concluding chapters themselves: "We do not contend that there is no relationship between these markets, but we feel certain that interest rates are only one of many factors which determine the course of prices in the stock market." "Interest is one of the costs which the speculator must carry, just as commissions are a cost, and changes in interest rates affect speculation just as changes in commission rates. Both are probably of negligible importance." To the extent that these conclusions carry weight with the speculative public, interest rates will lose their importance as a guide to stock speculation.

Should one care to ascertain just how fallacious these guides have been in the past, even when poorly used, the appendix with its index of stock prices and record of short-time interest rates over the past fifty years furnishes the material for investigation. The results of one such excursion into the appendix are as follows: Had a speculator decided in 1872 that hereafter he would purchase the stocks which are used in the computation of the index, the first month in which open market commercial paper rates averaged 4½ per cent, and sell them and go short an equal amount the first succeeding month in which they averaged 6½ per cent, closing that operation and buying when rates again averaged 4½ per cent, he would during the fifty years have lost 45 points, but as an offsetting item he would have gained 219 points. Had he used call loan rates and operated in the same manner, buying at 2 per cent and selling at 5½ per cent, he would have lost 41 points but gained 278 points. Had he not started until July, 1884, his losses would have been only 12 points while his gains would have been 266 points.

Truly one could better befriend the lambs of Wall Street than by destroying their faith in such pragmatic guides as these.

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WINFIELD W. RIEFLER

Population Problems in the United States and Canada, edited by Louis I. Dublin. Boston and New York: Houghton Mifflin Company. Published for the Pollak Foundation for Economic Research. 1926. xiii, 318 pp.

A symposium on population occupied the 1924 Christmas meeting of the American Statistical Association in Chicago. Nineteen of the papers are here published under the editorship of Dr. Dublin who, as president of the Association, arranged the meeting. His introductory chapter stresses the importance of the subject in this day of rapid changes in and regulation of the factors which affect population. Dr. Dublin warns us that we have almost reached what he calls "the danger line," but what some would call a normal state, of stationary population.

In Chapter II, E. B. Reuter presents the main facts of population growth in the United States. In Chapter III, "Natural Increase of Population," Warren S. Thompson concludes (1) that the rate of natural increase in the United States must decline, (2) that much of the improvement in living conditions, especially in cities, has been achieved by reducing the size of the family, and (3) that attention must be given to differentials in birth-rates. This author presents data to prove that the old American stock is dying out only in cities. Next, Hornell Hart studies urbanization. His data tend, though not conclusively, to show that migrants from country to city are more able and intelligent than the people who stick to the farm.

The relation of population to natural resources is naturally a central problem, and the next four chapters, dealing with this subject, are therefore of considerable interest. Moreover they contribute substantially to knowledge. A. B. Wolfe defends as an optimum population that for which, not the total income, but the per capita income of consumers' goods is a maximum. Here is a sharp contrast with the attitude which is content to allow population to drift on to the maximum attainable size. Our absurd statutes against contraceptive knowledge receive a bombardment from Professor Wolfe which should dismay the generals, priests and booster clubs. A corollary of the theory of the optimum is an argument for immigration restriction:

> When a country is at or near its optimum, an exclusion policy is entirely justifiable, for it is no country's duty to take in the surplus children of another. . . . Humanity, in every sense of the term, will be best served if each nation is held responsible for the poverty caused by its own sentiments and superstitions. This conclusion involves no assumption, such as frequently mars discussion and clouds the real issue, of hereditary inferiority on the part of any race or people.

Don D. Lescohier's paper, "Population and Agriculture," also argues for birth control. He sets forth that agriculture throuhgout the world is in a stage of

diminishing returns, and agrees with the 1923 *Year Book* of the Department of Agriculture that if we are to have any such population as 350 million it must be by considerably lowering our standards of living.

Alonzo E. Taylor considers item by item the problem of food for Pearl's estimated 175 million persons in the United States in 1980. Meat consumption is important here because people get much more food from a given land area when they eat its crops directly than when they eat animals which eat the crops. Assigning 3,500 calories per day to each person as a luxuriously adequate ration, Dr. Taylor finds that with reduction of our annual meat consumption per capita from 164 to 120 pounds, a figure between those of Germany and Great Britain before the war, the means of effecting the necessary increase in food production are at hand. Moderate increases in farm acreage and efficiency, abolition of scrub cattle and poor strains of grain, cessation of food exports, and the replacement by inedible substances of certain fats and other edible products now used industrially will, he calculates, keep the American dinner table plentifully supplied. But a danger is seen in the increasing demands for wood, paper, and fibers, which compete with crops and forage:

> When the ultimate population of the United States reaches 200 million, one does not see how the country can raise the desired primary foods, domesticated animals, lumber, paper, and fibers. But it is not the dietary standard that makes the pressure; it is, instead, the standards of our social and industrial living.

It might seem far-fetched to say that our forest fires and advertising campaigns by depleting great wooded areas will require more meatless days for our grandchildren; but the inference is plain.

Discussing "Mineral Resources for Future Populations," F. G. Tryon and Lida Mann find the fundamental difficulty in the supply of energy. All minerals must eventually be mined under conditions of sharply rising costs; but this would not be extremely serious if an abundant supply of cheap power could be depended upon. With petroleum and firewood disappearing, water power being an inadequate though valuable substitute, the main reliance must be on bituminous coal.

Henry Pratt Fairchild writes on "Racial Composition of the Population," warning against the irremediable though unproved evils of hybridization. But Ales Hrdlicka in the next chapter concludes from his biometric studies that immigration has not been altering the American type for the worse either physically or mentally. Next, W. W. Husband considers immigration policies, advocating relaxation of restrictions in favor of skilled workers of kinds specially needed. R. H. Coats describes Canada's program of encouraging agricultural immigration, a policy prompted by the needs of such great diminishing-cost industries as railroads and government. Alexander Goldenweiser assails "the Nordic myth," the hasty Americanizers, and the more extreme eugenists and psychologists. He appeals for toleration and appreciation of diverse cultures within the country.

George Soule opens the group of papers on Population and Labor Supply by calling attention to our great waste of man-power by unemployment, labor turnover, industrial accidents and ill health. The "labor shortage" resulting from restriction of immigration can be more than filled by reduction in these items, he

holds, and large increases in per capita production are attainable by improved management. Bryce M. Stewart discusses measures for regularizing employment and suggests a linkage of official city, state, national and international employment clearing houses connected with the immigration offices. The latter would have power to admit needed aliens of particular occupations upon requisition by employers. The plan has many advantages, but must face a difficult problem in definition. When is new labor "needed"? A requisition for a batch of immigrants may serve an employer as an alternative to an increase in wages or as a means of lowering them. The immigrants might be said to be "needed" in the first case and not in the second; but such a position is untenable.

A chapter by Bessie Bunzel and Robert Vane deals with the effect of the employment of women on population. The increase in such employment and its effect on birth- and death-rates are traced statistically. One interesting conclusion is that a large excess of males in a community leads to such competition for wives that the age of women at marriage is lowered and the birth-rate increased. Where females are in excess marriage is postponed, with a diminution in the ultimate family of about one-third of a child per year of delay.

The idea that civilization with its medicine and sanitation is interfering with natural selection and so causing the race to deteriorate receives a striking answer at the hands of Samuel J. Holmes. People whom science saves from death by accident or germ diseases die later of something; Professor Holmes shows that many of them die of constitutional disorders which are shrewder than germ diseases in selecting the unfit for elimination. Moreover, he points out, the excessive fecundity of the unintelligent is more or less compensated for by the high mortality of their children.

Paul Douglas argues for the graduation of wages according to number of dependents. To meet the objection that this would encourge over-expansion of population he relies first upon keeping the allowance per child less than the child's cost, and second upon birth control along with the emancipation of women.

R. A. MacIver surveys the prospects of changes in marriage-, birth- and death-rates, and concludes that a rather slight reduction in birth-rates will produce a stationary population. Emphasizing the need of reversing the present ratio of the fertility of the unintelligent to that of the intelligent, Professor MacIver urges a freer dissemination of birth-control information.

One chapter which might have been expected to appear in such a comprehensive book is missing: there is no discussion of the logistic curve, or indeed of any systematic method of interpolating or extrapolating to estimate population.

Inevitably a book by numerous authors has a journalistic air and leaves no unified impression. In the present case there is compensation in the general quality of the papers and their systematic arrangement.

HAROLD HOTELLING

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Profits, by William Trufant Foster and Waddill Catchings. New York: Houghton Mifflin Co. 1925. xxii, 465 pp.

This book falls into two parts which at first sight seem not to fit together; perhaps because the first appears as a defense of profits against certain common attacks, while the second is a merciless attack from the authors' own standpoint. The first part is a development of the risk theory of profits; while the second sets forth the theory of business depressions as due to the failure of business to divide to consumers an amount of purchasing power equal to the value of the goods turned out (at prevailing prices). The responsibility for this failure is jointly shared between the making of profits and the fact of saving, including not merely the "plowing back" of profits into the business, but individual savings as well.

The argument thus takes the form of presenting two sides of a dilemma: the necessary and serviceable character of profits and savings, and their inevitable effect in producing industrial depressions. In the first part, presenting the risk theory of profits, there is a strong development of the causal position of risk, marred perhaps by a too exclusive emphasis on this factor, and on the dependence of risk on consumers' freedom of choice. Only communism, by abolishing consumers' freedom and rationing all supplies, could abolish industrial risks. However, it is elsewhere stated that under communism we should have all the risks that now perplex us and some in addition.

Income from ownership is upheld on the ground that: "All present rewards for mere owning are in fact based on past rewards for doing"; an argument which might show some weakness if one pursued the question whether owner and doer are always the same or if not, just how the owner receives his claim. On the other hand, the authors make a valid point of much import in claiming that knowledge is essentially a public utility and may be so handled as to decrease materially the burden of economic uncertainty.

But this portion of the book is incidental, as indicated by the fact that the authors' summary of their argument makes no mention of it. The main thesis of the book is the insufficient-distribution theory of business depressions. This theory is developed by a series of hypothetical illustrations, starting with rigid and arbitrary assumptions and later attempting to see what difference, if any, is made by substituting more realistic conditions. One of the key assumptions is that, while the wages paid in any production-period are spent in that period, the dividends earned are divided and spent in the succeeding period. Another crucial premise is that the product made in one period is sold in the next; which is taken to mean that the earnings of any one period are limited to the receipts from the sale of goods produced in the previous period. An increase in current production, then, cannot be accompanied by a simultaneous increase in physical volume of sales. And since the increased productive activity typically involves increased distribution of wages; and since wages are, by assumption, spent in the period in which they are received, the result is an increase in prices which cannot, in the nature of the case, be maintained unless artificially supported by repeated inflations of the currency.

Another basic premise is that the whole machinery of production is integrated, so that the only "product" marketed is the ultimate output of consumption

goods; capital goods not being treated as part of the marketable product. While this assumption is, of course, legitimate for certain purposes, it requires the most careful handling to avoid unwarranted conclusions. A partial departure from it is involved in the assumption that profits may be locked up in unsold goods. This would not be in accord with good accounting practice for a consolidated concern; but in the actual non-integrated industrial system profits may be recorded on the sale of goods to producers or dealers, such profits representing an anticipation of sales to ultimate consumers. Thus the industrial system as a whole may anticipate profits, though good accounting would not permit a single concern to do so.

Any increase of profits at the expense of wages involves a lag in the spending of the funds, with the result that the profits can only take the shape of unsold goods, which cannot later be disposed of in addition to the usual output (at current prices), since there is not enough money to buy both. And savings, while their first effect is a mere shift of purchasing power from Consumers' to producers' goods, result in a similar surplus of unsold goods when the ultimate increased product comes on the market. The effect of installment buying is pictured as a limited anticipation of future buying power, where a progressive increase is needed to cure the evil. And much the same is said of extensions of bank credit.

No specific remedy is put forward; but it is clearly indicated that the necessary cure must lie in increasing the fund of purchasing power spent for ultimate products. Mere inflation is not sufficient: it must be guided into the proper channels to preserve a proper balance betwen production and consumption.

As a prize of $5,000 has been offered for the best adverse criticism of this book, an attempt at evaluation in a brief review would probably be premature. The main weaknesses will undoubtedly be found in the assumptions of the hypothetical cases, such as the ones suggested above. The most serious admission of the weakness of these hypothetical cases consists in the statement that cases could be so constructed as to permit increased savings and increased output to go hand in hand indefinitely, if everything were timed and proportioned correctly; but that in actual industry this correct timing and proportioning never takes place. Hence the actual result is always an inflation of prices, followed by an inability to dispose of the goods industry is able to produce without a fall in prices, which itself always brings about losses and stagnation of productive activity. This seems to involve a radical shifting of ground from the argument of the hypothetical cases. One might also note that "profit" changes its meaning from the first half of the book, where it means gains above interest, has no standard rate, and is quite likely to aggregate a minus quantity; to the second half of the study, where it means the total net earnings of a business, has a standard rate, and aggregates a large amount; this being essential to the argument.

The book is far from being a bit of obviously fallacious amateur economics and cannot be dismissed as such. It shows internal evidences of an earlier form of theory, modified at many points by the criticisms of "regular" economists without abandoning the original central thesis. As to whether the modifications are sufficient, and are consistent with the retention of the main framework of the basic theory, the results of the prize contest will probably do much to reveal.

Columbia University. J. M. Clark

Massachusetts Commission on Old-Age Pensions, Final Report, 1925. Boston: State Printer. 280 pp.

This is the final report of the fourth commission appointed under resolves of the Legislature of Massachusetts for the purpose of investigating and reporting upon the subject of old-age pensions.[1] It had been preceded by a preliminary report dealing chiefly with civil pensions. Hence the report under review deals exclusively with the problem presented by the aged poor who have been in private life. Its value lies in the fact that it presents the results of the most inclusive investigation made by any state commission in this country and that it contributes much to a statement of the problem. Incidentally it will be of service in revealing the superficiality and inaccuracy of much that has been said in print on the subject.

From data obtained from the larger employers, benevolent homes, public almoners, charity organizations, and others, and from a canvass of a representative number of the aged, the Commission undertook to measure the problem and to state it in terms that would be most helpful in connection with the more important proposals for old-age pensions. The number of the aged, the number of the dependent and of the non-dependent, property owned by them and their incomes, their occupations past and present, whether or not those in almshouses, benevolent homes, etc., could live outside, relatives able to support the aged fully or in part, in fact all important things necessary to state the problem with a fair degree of accuracy were covered in the investigations made. While the statistician may find some things which will not meet with his entire approval, he will recognize that the Commission made excellent use of the statistical method. The data are set out in tables and text in such manner that they can be readily brought to bear upon specific proposals for legislation.

A very considerable number of the aged have military or civil pensions and many are retained in employment or are cared for by their last employers through pensions or in less formal ways; many have sufficient property or income to provide for themselves and many of those who have little or no property or income have relatives who can assist them; a considerable number are in state and local institutions and most of these would have to remain there because they require institutional care. Yet the Commission found that if pensions were limited to citizens 70 years of age and over who have resided in Massachusetts for at least fifteen years and whose property amounts to less than $3,000 with an income of less than $365 per year, some 34,600 would qualify for grants under a non-contributory old-age pension law. If the qualifying age was fixed at 65 the number of pensioners would be, in round numbers, 51,000. The total number of persons over 70 is estimated at 113,000, over 65 at 225,000. Thus it would be expected that some 22.6 per cent of those over 65 and some 41.6 per cent of those over 70 would qualify for pensions, if extended to citizens who have resided in the state for fifteen years or more.

While agreeing unanimously that the problem revealed called for additional public action, the Commission, as might have been expected, disagreed as to the

[1] Reports had been made by commissions in 1910, 1914, and 1917. Moreover, the Massachusetts Census in 1915 had investigated and reported relative to aged and dependent persons in the state.

advisability of adopting an old-age pension law. Such a law was recommended by three members of the Commission; the other two submitted a minority report. In this minority report all of the arguments usually advanced against non-contributory pensions (charity, not adjusted to needs, undermining thrift, expensive and likely to become more so because of political pressure, presenting administrative problems, etc.) are very well set forth. The alternative suggestion made is improvement in the existing types of institutions caring for the aged.

The report contains a number of useful appendices, among them those on old-age pensions in other states, and on old-age pensions in foreign countries.

H. A. Millis

University of Chicago

A Manual of Problems and Tables in Statistics, with Notes on Statistical Procedure, by Frederick C. Mills and Donald H. Davenport. New York: Henry Holt and Company. 1925. v, 203 pp.

This volume, a sequel to Professor Mills' excellent text, *Statistical Methods Applied to Economics and Business*,[1] is intended chiefly as a companion volume thereto. It should not be assumed, however, that a teacher would have difficulty in using it with any other good text—such as Day, Jerome, Chaddock, Secrist, or Crum and Patton.

There are two reasons for its easy use: (1) Problems are given in sufficient number and diversity to cover practically every part of the usual first-year work in statistics. They are so grouped in the body of the book, and the groups are so listed in the Table of Contents, as to make ready reference easy despite the lack of any index. The order of treatment in the textbook which happens to be used is therefore almost a matter of indifference to the teacher in selecting and assigning appropriate problems. (2) But almost one-half of the book does not consist of problems at all: these are confined to Part III ("Problems and Exercises") and, virtually, Part IV ("Selected Time Series"). The other six parts provide the student with "pointers" and reference matter which are entirely independent of the choice of text, and probably outweigh in value the problems themselves. In respect to the reference sections particularly, a book agent could repeat with truth the time-worn slogan that the volume does "fill a long-felt want."

Part I offers to the student valuable "Notes on the Conduct of a Statistical Investigation." Most wise teachers will, to be sure, avoid inflicting this material upon a student before he is well seasoned in what may be broadly termed the *apparatus* of statistical procedure—charts, tables, averages, coefficients, and the like.

As Mr. Edwin W. Kopf, the present reviewer, and others sought to demonstrate against opposition at one of the Round Tables of the Chicago meeting of the Association in 1922, the order in which the statistical investigation should be conducted is not the best order in which the subject of statistics should be taught. This statement, applicable to several other steps in investigation, is peculiarly

[1] Published in 1924. Reviewed by Melbourne S. Applegate in this Journal, Vol. XX, pp. 295-296, June, 1925.

applicable to the initial step of planning. Mills and Davenport evidently realize the truth of the statement, both in general and in particular; otherwise Mills would not have employed his unusual (and highly effective) order of chapter arrangement in the text-book.

But after a half-year of seasoning on such a text the student will be really ready to visualize, with both interest and effect, the statistical investigation as a whole, including the plan stage. The authors do well to reserve this subject for the sequel volume, and thereby avoid diverting and confusing the student's mind as he studies his text; but its appearance as the *first* part of the new manual does not imply that it should be taught before proceeding to other parts thereof.

The "Suggestions for a Statistical Library," contained in Part II, are by no means complete, nor are the titles followed by critical notes adequate for guiding a student interested in building up a small library. Hints from the teacher will be found necessary for this purpose. However, no source listed by the authors could be omitted from the library of an institution which expected to give first-rate instruction in the field; and the authors—perhaps wisely—are trying to help the institution more than the student in this respect.

Part III contains "Problems and Exercises" which, like the "Selected Time Series" in Part IV, are free from error to an extent which is surprising in a volume of this kind, still in its first edition. Class use of them for a year has convinced the reviewer of their adequacy for nearly every usual purpose, although the teacher's own file of problems must also be drawn upon to introduce variety, especially if classes be large and some of their members prone to "collaborate" excessively!

If a teacher uses Greek symbols to any great extent, he quickly finds that they are truly "Greek" to many of his students. Mills and Davenport have not overlooked even the phonetics of the Greek alphabet in their very complete "Glossary of Symbols" (Part VI). The equally careful compilation of derived "Statistical Formulas" in Part B and the tables of logarithms, roots, reciprocals, and powers, with areas and ordinates of the normal curve of error in terms of x, go far toward making the manual a self-contained compendium of all the necessary materials for general first-year work which the printed page can by its nature supply.

The book is a fitting accompaniment to Mills' sound yet thoroughly teachable textbook. It is based upon the axiom that "in this field one learns by doing, and only by doing." Taken together, these two companion volumes apparently constitute the most effective teaching apparatus for elementary statistical work which the present reviewer has ever seen.

W. A. Berridge

Brown University

The Oil Industry and the Competitive System: A Study in Waste, by George Ward Stocking. New York: Houghton Mifflin Company. 1925. 27 figs.; 323 pp.

This book, the winner of the first prize in the 1924 Hart Schaffner and Marx essay contest in economics, deals with a subject that is very much in the public eye. The phenomenal growth in the demand for gasoline, the repeated warnings by geologists that the supply of crude petroleum is limited, and finally the investigation now nearing completion by the Federal Oil Conservation Board, the preliminary findings of which have already been given to the public, have all combined to draw attention to the petroleum industry and the manner in which the petroleum reserve is being exploited.

The present study is broken rather sharply into two parts. The first portion is historical and deals with the growth of the old Standard Oil Company, the methods employed by it in attaining its position of so-called monopoly, the litigation leading to its dissolution in 1911, the effectiveness of this action in fostering competition, and finally the present position of the former components of the Standard Oil Company. This review of the subject is carefully made and presented; but reflects the conclusions reached by the Bureau of Corporations, the Federal Trade Commission, and Miss Ida Tarbell in her *History of the Standard Oil Company*. The author finds that, despite the efforts of the Government to force competition into the petroleum industry, monopoly persisted; and only at the very end of the historical section does he suggest, after noting that a marked degree of competition has developed in the industry in general and between the Standard Oil companies in particular, that "the interdependence of the various Standard companies is approaching an end. It may be," he adds, "that the force of economic circumstances will yet make the oil industry genuinely competitive and free."

The second part of the book is devoted to a description of the production, transportation, refining, and marketing of oil, with particular reference to the manner in which competitive efforts lead to "waste." This section is based in part upon the experience of the author as an employee of the geological department of an oil company in the oil fields of North Texas during 1919, and in part upon a careful survey of the technical and economic literature of the subject. It is pointed out that the small land holdings prevalent in practically all oil pools are the cause of rapid and uneconomic drilling, and that this circumstance is responsible for an undue dissipation of gas pressure and consequently a smaller recovery of oil than would be obtainable under less competitive conditions. Numerous illustrations of competitive drilling are adduced, both from the author's experience and from the literature. The results of competition are then followed through the other departments of the oil business, and the shortcomings of competitive efforts are catalogued and stressed. This portion of the book is interestingly presented and carefully documented; but it perhaps overcolors the subject by failing to present the technologic improvements that have taken place in the past few years—improvements that have almost revolutionized the industry.

In conclusion, the author devotes a chapter to the question, Is there a way out?

He thinks there is. His remedy is: Encourage the work of the United States Geological Survey and the United States Bureau of Mines; appoint a conservation board to study the problem (this has subsequently been done); and institute "a comprehensive system of federal regulation and control of petroleum production." As to the last mentioned remedy: by what warrant the Government can be expected to be competent in such an undertaking in the face of experience that "such direct interference or control as our Federal Government has seen fit to exercise through the anti-trust laws has tended to encourage rather than check wasteful methods of exploitation," the author does not state.

JOSEPH E. POGUE

RECENT LITERATURE

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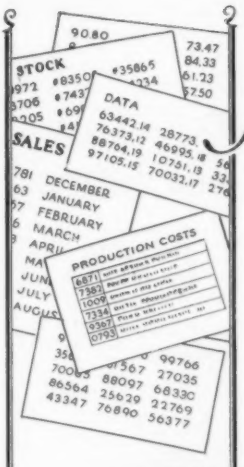

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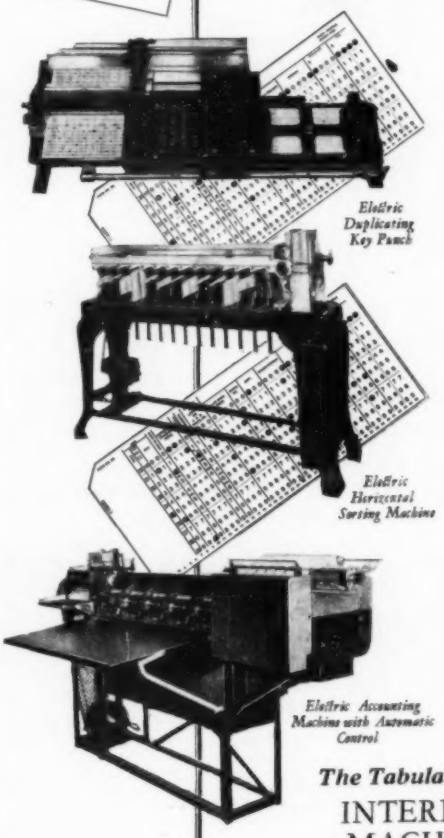

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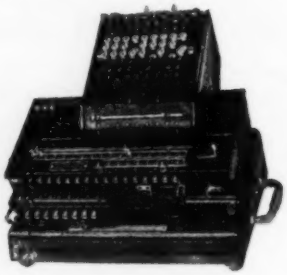

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